

Naturalness and Higgs decays in the MSSM with a singlet

Spencer Chang,^a Patrick J. Fox^b and Neal Weiner^a

^a*Center for Cosmology and Particle Physics, Dept. of Physics, New York University
New York, NY 10003, U.S.A.*

^b*Theoretical Physics Group, Lawrence Berkeley National Laboratory
Berkeley, CA 94720, U.S.A.*

E-mail: sc123@cosmo.nyu.edu, PJFox@lbl.gov, nw32@nyu.edu

ABSTRACT: The simplest extension of the supersymmetric standard model — the addition of one singlet superfield — can have a profound impact on the Higgs and its decays. We perform a general operator analysis of this scenario, focusing on the phenomenologically distinct scenarios that can arise, and not restricting the scope to the narrow framework of the NMSSM. We reexamine decays to four b quarks and four τ 's, finding that they are still generally viable, but at the edge of LEP limits. We find a broad set of Higgs decay modes, some new, including those with four gluon final states, as well as more general six and eight parton final states. We find the phenomenology of these scenarios is dramatically impacted by operators typically ignored, specifically those arising from D-terms in the hidden sector, and those arising from weak-scale colored fields. In addition to sensitivity of m_Z , there are potential tunings of other aspects of the spectrum. In spite of this, these models can be very natural, with light stops and a Higgs as light as 82 GeV. These scenarios motivate further analyses of LEP data as well as studies of the detection capabilities of future colliders to the new decay channels presented.

KEYWORDS: Higgs Physics, Supersymmetry Phenomenology.

Contents

1. Introduction	1
2. Naturalness in the MSSM	2
3. Summary of LEP limits on Higgs	5
4. Singlets and the MSSM	6
5. Scenarios	14
5.1 Higgs Decays Through a Single Stage Cascade Into b Quarks	16
5.2 Higgs Decays Through a Single Stage Cascade (without b's)	19
5.3 Higgs Decays Through a Two Stage Cascade	23
6. Phenomenology and Benchmark Point Summary	25
6.1 Possible exclusions in existing data	26
6.2 Future experiments	28
7. Summary and Conclusions	28
A. Trilinear couplings	30

1. Introduction

Since 1934 when Fermi wrote down his theory of the weak interactions, we have singled out the weak scale as an interesting scale for new physics. Within the last two decades, we have finally reached this energy scale at LEP and the Tevatron, and with the LHC we should be able to probe this scale thoroughly.

The most pressing question at the weak scale is the origin of electroweak symmetry breaking (EWSB). Within the Glashow-Salam-Weinberg theory of weak interactions, it is broken by a fundamental scalar doublet of $SU(2)$, the Higgs field. In this framework, all precision quantities are calculable and agree with present experimental limits.¹

The major theoretical shortcoming of the standard model (SM) is the question of the stability of the Higgs mass. Scalars in general acquire quadratically divergent mass, suggesting that new physics should cut off this divergence and appears near the weak scale. Numerous solutions to this theoretical problem have been proposed, most notably supersymmetry (SUSY).

¹We are of course neglecting dark matter here, neutrino masses, the LSND anomaly and the NuTeV anomaly, all of which are beyond the scope of this paper.

2. Naturalness in the MSSM

In this paper we shall focus on supersymmetric solutions of the hierarchy problem. In these theories the introduction of superpartners cancels the quadratic divergences of the standard model with loops of opposite statistics particles, leaving only logarithmically divergent contributions to the Higgs mass, proportional to the SUSY breaking soft mass parameters.

Despite its excellent success in controlling divergences from very high scales, in recent years SUSY has become far more constrained, with superpartners pushed to higher scales, resulting in tunings which are typically $O(4\%)$ in the minimal supersymmetric extension of the standard model (MSSM).² The reason for this is quite simple: the success of the LEP program has pushed the lower limit on a SM Higgs boson above 114 GeV, a limit which applies in a large region of the MSSM parameter space. In the MSSM, the Higgs has a mass which has a tree level upper limit of $m_Z = 91$ GeV. To achieve a mass of 115 GeV, one must invoke large radiative corrections due to (s)top loops. At one loop, the correction to the Higgs mass is [2]

$$\delta m_h^2 = \frac{3}{4\pi^2} v^2 y_t^4 \sin^4 \beta \log(m_{\tilde{t}_r} m_{\tilde{t}_l} / m_{\tilde{t}}^2), \quad (2.1)$$

requiring a stop mass of roughly 500 GeV. Unfortunately, such a large stop mass also feeds into the soft mass squared of the Higgs,

$$\delta m_{H_u}^2 = -\frac{3y_t^2}{8\pi^2} (m_{\tilde{t}_l}^2 + m_{\tilde{t}_r}^2) \log(\Lambda / m_{\tilde{t}}), \quad (2.2)$$

where Λ is the scale where the stop masses are generated. Even for low scale mass generation such as gauge mediation, where $\Lambda \gtrsim 16\pi^2 m_{\tilde{t}}$, this results in a radiative correction to the up-type Higgs soft mass squared of $O((400 \text{ GeV})^2)$. This must then be cancelled against a positive mass squared, for instance via a μ -term, in order to achieve the appropriate vev, $v = 174$ GeV. This cancellation must be tuned at the level of 4% in order to achieve this. If the stops are heavier, or the mediation scale is higher, the tuning becomes worse.

The root of this problem is the small quartic of supersymmetric models, which is fixed by the electroweak gauge couplings. There are many proposals to enlarge the quartics, for instance by adding terms in the superpotential [3–10], with non-decoupling D-term quartics [11–13], or through strong dynamics at an intermediate scale [14–16].

New Higgs Decays and Naturalness

An alternative approach is to evade the LEP limits indirectly. For instance, if the Higgs field decays in a non-standard way, LEP may not have been as sensitive to the decay. This is not as trivial as it may appear, in particular because limits on invisible Higgs decays are nearly as strong as those of b-quarks. If these non-standard decays are present in nature, there must be new states lighter than the Higgs boson, which themselves decay

²For an excellent discussion of the details of fine tunings in SUSY models, see [1].

into ordinary SM fields. The simplest possibility is the presence of an additional singlet superfield to which the Higgs can decay, and which then decays into SM particles, such as b-jets. In this vein, it has been recently argued that within the NMSSM, where a singlet field S acquires a vev to supply the μ -term of the Higgs sector, certain non-standard decays are possible, leading to more natural theories [17].

However, there is still a strong need for additional analyses, for many reasons.

- New combined limits from LEP [18, 19] exclude most of the generic parameter space of these models. Typically, the decay $h \rightarrow 2a \rightarrow 4b$ has been used to allow lighter Higgses. However, new LEP combined analyses have basically raised the minimum Higgs mass for this process to 110 GeV, nearly as strong as the limit on the standard model Higgs. This raises the question of whether other non-standard decays can occur generically, to which LEP analyses would have been less sensitive. We will discuss these limits further in the next section.
- The NMSSM is not a fully general scenario. In the NMSSM, the singlet acquires a vacuum expectation value to generate a μ -term. One typically requires that this be the true vacuum of the theory, and additionally makes assumptions of the form of the theory in the ultraviolet (UV). All of these things can distract from — and are beside the point of — the basic phenomenological questions relating to Higgs decays.
- The sensitivity of m_Z to UV parameters is not the only measure of naturalness in these theories. Although a significant reason to consider these decays is precisely to ameliorate this tuning, one often ends up with additional tunings in order to evade experimental limits. Thus, while m_Z may be relatively natural other elements of the spectrum may be highly tuned, and necessary, to evade experimental limits.
- The parameter space of new operators has not been fully explored. In the MSSM, the full set of soft breaking operators is considered. However, in adding a singlet to the theory, one can consider both the effects of D-terms in the supersymmetry breaking sector, and non-degenerate soft masses for the scalar and pseudoscalar components of the singlet, both of which can have significant phenomenological consequences.
- The sensitivity of the decays of the pseudoscalar to the presence of new fields has essentially been ignored within the context of supersymmetric theories. Such fields can naturally induce the decay of the pseudoscalar to two glue jets, which have far weaker constraints.

There are two important additional points to be made with regard to naturalness. The first is: what do we mean by tuning in the scenarios we are considering? Since we are interested in the low-energy phenomenological theory, it is impossible to quantify the sensitivity of m_Z to the UV parameters. However, we know that the tuning of m_Z arises in general due to the large values of the stop squark masses. Therefore, we shall use the stop masses as a proxy for this tuning. However, there is typically a tuning necessary to achieve the proper spectrum for the lighter scalar states, and this is usually the most severe

tuning in the models. Therefore, we shall quote both stop masses, as well as the scalar mass spectrum tuning for every model considered.

The second point is: how natural is it to include a soft supersymmetry breaking operator, without an associated superpotential operator? In particular, how reasonable is it to include a trilinear scalar potential A-term operator without associated Yukawas? This question has been studied previously in the context of sterile neutrino masses in supersymmetry [20, 21]. In fact, if there is a SUSY breaking field which carries a charge, whether R-charge or Peccei-Quinn charge, it is quite natural for these operators to appear without an associated superpotential term. The converse, however, is not true. The presence of the superpotential term will radiatively generate the soft term at a minimum at the loop-suppressed level. Consequently, we will insist on technical naturalness, that these soft operators are present at least at this small level.

In lieu of these points, we will pursue a phenomenological study of the effects of singlets on the decays and properties of Higgs bosons. We will focus on decay modes that have not been considered previously, including final states with 6 and 8 particles. We will often be studying situations when both the new scalar and pseudoscalar states are lighter than the Higgs, and the Higgs is light enough to have been produced through Higgs-strahlung at LEP, although we will consider other scenarios. By also focusing on the relevant parameters, we hope to clearly elucidate the effects of mixing of the singlet with the Higgs, and the effect on the mass of the Higgs boson.

Since this paper has both basic phenomenological points, as well as points related to model building, we have attempted to lay out this paper so that one who is interested principally in the phenomenology can still learn the relevant points. Sections headed “Model Building” are independent, and the paper can largely be read without those.

The layout of the paper is as follows: in the next section, we will review existing limits on Higgs decays, both for SM-like and non-SM-like Higgs bosons. In section 4, we will discuss the effects of singlets on Higgs physics. First, we show how non-standard decays into scalars can arise and dominate the Higgs decay width. We will also point out how non-standard decays allow a larger mixing with the Higgs, this mixing can raise the mass of the Higgs boson considerably without resorting to radiative corrections. In its “Model Building” subsection, we present the relevant operators that induce the Higgs mixing and decays. In section 5, we will sketch out the spectra and non-standard decay modes which are consistent with existing LEP bounds, and lead to a more natural parameter space of the theory. In its “Model Building” subsections, we discuss model realizations of these different spectra and decay scenarios. In doing so, we can assess more simply the degree of tuning required to satisfy the experimental constraints. In section 6, we summarize the presented benchmark points and their relevant phenomenology for future collider experiments. In section 7, we discuss additional directions worthy of further investigation and conclude. In particular, we suggest some new analyses on LEP data which might be useful when considering these scenarios. Finally, in appendix A some calculational results for scalar trilinears are summarized.

3. Summary of LEP limits on Higgs

Most particle physicists are familiar with the LEP2 95% CL limit on the SM Higgs of $m_h \geq 114.4$ GeV, but LEP has also produced a multitude of other limits on Higgs physics.

The limits that are most applicable to this work are the 95% CL limits on the so called ξ^2 parameters (often also referred to as C^2). We are analyzing a SM-like Higgs, which means that Higgs-strahlung production of the Higgs is close to the SM rate.³ Then depending on the assumed process of Higgs decay $h \rightarrow X$, the defined limits are on $\xi^2 \equiv \frac{\sigma(e^+e^- \rightarrow hZ)}{\sigma(e^+e^- \rightarrow hZ)_{\text{SM}}} BR(h \rightarrow X)$. We now list the different limits that LEP has analyzed, based on Higgs-strahlung and the given Higgs decay process (note: all stated mass limits assume $\xi^2 = 1$):

- **Model Independent Decays:** This is the most conservative limit on the Higgs boson. It assumes that the Higgs is produced with a Z boson and looks for electrons and muons that reconstruct to a Z mass, while the Higgs decay process is unconstrained (by theory or the event analysis). The only study of this sort is done by OPAL giving a limit of $m_h \geq 82$ GeV, see figure 11 of [22]. Unfortunately, no other collaboration has released such an analysis.
- **Standard Model Higgs:** LEP-wide limits on the SM Higgs are given in figure 10 of [23], requiring $m_h \geq 114.4$ GeV. This study also includes the strongest limits on $h \rightarrow b\bar{b}, \tau\bar{\tau}$ rates.
- **Invisible Decays:** In this analysis, the Higgs is assumed to decay into stable (on collider length scales) neutral particles. The implication is that non-standard Higgs decays have to primarily decay into visible particles. Both L3 and DELPHI have performed such an analysis [24, 25], but the most stringent constraints are from an older preliminary LEP-wide analysis that has a limit $m_h \geq 114$ GeV, see figure 4 in [26].
- **2 Photon Decays (aka Fermiophobic):** Fermiophobic typically means a Higgs with standard couplings to gauge bosons, but suppressed couplings to fermions, allowing decays into WW^*, ZZ^* as well as photons. If all decay modes are open, there is a limit of $m_h \geq 109.7$ GeV, while decays exclusively to two photons have a limit of $m_h \geq 117$ GeV. See figure 2 of [27], which is the LEP-wide analysis.
- **2 parton hadronic states (aka Flavor-independent):** In this type of analysis, any two jet decays of the Higgs are allowed. The analyses use the jets that are least sensitive to the candidate Higgs mass and details of the Z decay. Each LEP collaboration has done a study [28–31]. However, the strongest constraint is the preliminary LEP-wide analysis of $m_h \geq 113$ GeV, see figure 2 of [32].

³More generally, we will usually be taking the decoupling limit, where the lightest Higgs state is produced by Higgs-strahlung, but not in associated production with the CP-odd A^0 . Thus, constraints from Higgs-strahlung are the main concern in these scenarios.

- Cascade Decays: These are the constraints that are most relevant for the present study, where cascade decays mean that the Higgs decays into two scalars ϕ and those scalars decay into X (i.e. $h \rightarrow 2\phi \rightarrow 2X$). OPAL [33] and DELPHI [34] looked at b decays ($X \equiv b\bar{b}$), see their figure 12's and a new LEP-wide analysis [18, 19] has constrained both b and τ decays, with the exclusion plots given on page 8 in the first reference. For $h \rightarrow 2\phi \rightarrow 4b$ the limits are now 110 GeV for a Higgs produced with SM strength. For other intermediate scalar decays, $\phi \rightarrow 2g, c\bar{c}, \tau\bar{\tau}$, the best model independent exclusions are from OPAL's analysis when the mass of the scalar is below $b\bar{b}$ threshold, which is given in figure 7 of [35] (note: the analysis is restricted for Higgs masses in the range 45 – 86 GeV).

4. Singlets and the MSSM

As we have previously said, the simplest extension of the MSSM is to add a SM singlet superfield S , containing a CP-even scalar s , and a CP-odd pseudoscalar a .⁴ The dominant phenomenological effects of this new field are:

- New decays for the Higgs boson. If one or both of the new states are lighter than half the Higgs mass, decays $h \rightarrow 2s$ and $h \rightarrow 2a$ are possible, followed by $s \rightarrow X$ or $a \rightarrow X$.
- Light states can mix with the Higgs boson. If a light state mixes with the Higgs boson, it can push the mass of the Higgs boson up without large radiative corrections. This is at the cost of having a new light state which can be produced through brehmstrahlung off a Z , for which there are stringent constraints.

We first show that non-standard decays can easily dominate over SM decays. In the case of cascade decays, for a Higgs produced with SM strength through Higgs-strahlung, the new process must have a width at least⁵ $\Gamma_{h \rightarrow 2a} \gtrsim 4 \times \Gamma_{h \rightarrow 2b}$. With a term in the Lagrangian

$$\frac{cv}{\sqrt{2}} ha^2 \tag{4.1}$$

the width to two scalars is [36]

$$\Gamma_{h \rightarrow 2a} = \frac{c^2 v^2}{16\pi m_h} \left(1 - \frac{4m_a^2}{m_h^2}\right)^{1/2} \tag{4.2}$$

which one can compare with the dominant width of the Higgs to b quarks (in this mass range)

$$\Gamma_{h \rightarrow b\bar{b}} = \frac{N_c m_b^2}{16\pi v^2} m_h \left(1 - \frac{4m_b^2}{m_h^2}\right)^{3/2} \sim .003 \text{ GeV} \left(\frac{m_h}{100 \text{ GeV}}\right). \tag{4.3}$$

⁴More precisely, the CP-even properties of a are not determined until its interactions and mixings are given. For example, it can pick up a CP transformation if it mixes with a scalar with fixed CP properties (the CP-even h^0, H^0 or CP-odd A^0) or couples to fermions/gauge bosons in a certain CP fashion. On the other hand, in all cases s mixes with h^0 and thus is always CP-even.

⁵This condition is not sufficient if the scalar a decays into b quarks, as will be discussed later in section 5.1.

Taking into account higher order effects, a more accurate approximation for the Higgs decay width to SM particles in the mass range of interest is (using HDECAY [37])

$$\Gamma_{h \rightarrow SM} = (2.34 \times 10^{-5} m_h + .206 \text{ MeV}) \quad (4.4)$$

For a 100 GeV Higgs, in order for scalar decays to sufficiently dominate, one must have

$$\frac{cv}{\sqrt{2}} \gtrsim 5 \text{ GeV} \quad (4.5)$$

The required size of the trilinear indicates that non-standard cascade decays can easily dominate over standard Higgs decays.

One can also study the effects of mixing with a singlet on the Higgs mass in a model independent fashion. In the presence of mixing, the mass eigenstates \tilde{s} and \tilde{h} will be related to the interaction eigenstates through a mixing matrix

$$\begin{pmatrix} \tilde{s} \\ \tilde{h} \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} s \\ h \end{pmatrix} \quad (4.6)$$

Similarly for the mixing in the CP-odd sector, the mass eigenstates \tilde{a} and \tilde{A} are related to the interaction eigenstates by

$$\begin{pmatrix} \tilde{a} \\ \tilde{A} \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} a \\ A^0 \end{pmatrix} \quad (4.7)$$

If the MSSM mass of the Higgs is m_{mssm}^2 , then the mass eigenvalue after mixing with the lighter singlet is

$$m_{\tilde{h}}^2 = \frac{m_{mssm}^2 - m_{\tilde{s}}^2 \sin^2 \theta}{\cos^2 \theta}, \quad (4.8)$$

where $m_{\tilde{s}}$ is the mass of the mass eigenstate \tilde{s} . This increase in mass is only through mixing and not through radiative corrections.

In the subsection below, we discuss some of the operators which can be used in building models with certain phenomenologies. Those readers who are only interested in the descriptions of the phenomenology can proceed to section 5.

Model Building: New Operators with Singlets

The introduction of a singlet superfield allows us to introduce a number of new operators in the theory. These operators have the effects of introducing into the theory masses, mixing, and decays of the states. The operators are listed in table 1. We will describe here the effects of these operators when they appear singly. Clearly, the effects of the operators do not necessarily add linearly, particularly when vevs turn on, but it is useful to understand their effects individually. In many scenarios this is sufficient to understand the phenomenology and we will point out the situations where there are interference effects from multiple operators.

δ_s^2	$XX^\dagger(S + S^\dagger)_D^2$	mass for s
δ_a^2	$XX^\dagger(S - S^\dagger)_D^2$	mass for a
m_D	$(W'_\alpha W^\alpha S)_F$	singlino (Dirac) mass, mixes s and h allows $h \rightarrow 2s$ decays, allows $Z s$ production
λ_h	$(SH_u H_d)_F$	(with mu term) mixes s and h , s and a masses, singlino (Dirac) mass, allows $h \rightarrow 2s$, $h \rightarrow 2a$ decays
A_s	$(XS^3)_F + h.c.$	Real: (with (s)) mass for s $s \rightarrow 2a$ decays, 3 s coupling Imaginary: $a \rightarrow 2s$ decays, 3 a coupling
A_h	$(XSH_u H_d)_F + h.c.$	mixes s with H , a with A^0 , a decays (due to mixing) with mixing, allows $h \rightarrow 2s$, $2a$ decays (esp. large $\tan\beta$)
m_{CP}^2	$XX^\dagger(S + S^\dagger)(S - S^\dagger)_D$	Mixes s and a , allows a decays (due to mixing)
M_Q^{-1}	$(SQ\bar{Q} + M_Q Q\bar{Q})_F$	Allows s and a decays to gluons and photons

Table 1: Operators involving the Singlet

δ_s^2, δ_a^2 — **Scalar and Pseudoscalar Soft Masses**

The operators arising from Kähler potential terms, δ_s^2 and δ_a^2 , determine the spectrum of the theory by adding mass terms to the Lagrangian

$$\mathcal{L}_{\delta_s^2} = -\frac{\delta_s^2}{2}s^2 \quad \mathcal{L}_{\delta_a^2} = -\frac{\delta_a^2}{2}a^2. \tag{4.9}$$

Most previous analyses include equal soft masses for the scalar and pseudoscalar, although there is often no a priori symmetry reason to do so. All other scalar fields in the MSSM carry a gauged $U(1)$, forcing a degeneracy which is not necessary for singlets. Because our interests are phenomenological, we allow different masses, and thus more interesting spectra. In particular, this allows spectra of the form $m_h > 2m_s > 4m_a$, leading to events with many final state jets, b-quarks and photons, produced through cascades.

The tuning of these theories with singlets, when $m_h < 114$ GeV, is usually encoded in how tuned these additional masses must be in order to achieve a proper spectrum, and not in the sensitivity of m_Z or the masses of the stops. When we refer to these operators, a value of δ_s^2 or δ_a^2 of $\pm(20\text{GeV})^2$ refers to a contribution of $\pm(20\text{GeV})^2$ to the scalar or pseudoscalar mass squared.

m_D — **Supersoft Operator**

m_D is the so-called “supersoft” operator [38] and arises from a SUSY breaking spurion in the superpotential. This operator is particularly interesting, and often neglected. If the hidden sector has a $U(1)'$ which acquires a D-term expectation value, it has no effect in the MSSM, in that all soft operators can be generated by F-term breaking. On the other hand, in the presence of a singlet, D-term breaking becomes important through the introduction of this new operator [39]. The Lagrangian terms generated are

$$\mathcal{L} = \int d^2\theta \frac{W'_\alpha}{M} W_Y^\alpha S + h.c. \rightarrow -\frac{1}{2}(m_D s + D_Y)^2 \tag{4.10}$$

where D_Y is the usual hypercharge D-term, $D_Y = \sum_i g_Y q_i \phi_i^* \phi_i$, and $\frac{W'_\alpha}{M} = \theta_\alpha m_D$. Although it is supersymmetry breaking, it does not feed into the RG flow of any other soft

operators, hence the term supersoft. The principal effects are to produce a mass for s (but not a), and a coupling shh^* which is proportional to the new contribution to the s mass. It also generates a Dirac mass for the Bino with ψ_S equal to $m_D/2$. When we refer to a value of m_D of 20 GeV, this implies a contribution to the scalar mass squared of $(20\text{GeV})^2$, and the corresponding trilinear term.

This operator directly affects the physics of electroweak symmetry breaking and the properties of the Higgs. However, it is simple to perform the same minimization of the Higgs potential in the presence of this term. Let us consider the usual MSSM neutral Higgs potential with this contribution.

$$\begin{aligned}
 V = & (|\mu|^2 + m_{H_u}^2)H_u^2 + (|\mu|^2 + m_{H_d}^2)H_d^2 - (bH_uH_d + h.c.) \\
 & + \frac{g^2}{8}(H_u^*H_u - H_d^*H_d)^2 \\
 & + \frac{1}{2} \left[m_D s + \frac{g_Y}{2}(H_u^*H_u - H_d^*H_d) \right]^2 + \frac{\Delta_s^2}{2}s^2
 \end{aligned} \tag{4.11}$$

where Δ_s^2 encodes other contributions to the s mass, such as from δ_s^2 .

When electroweak symmetry is broken, there is a linear term for s , and so we shift s by an amount

$$s \rightarrow s + \frac{g_Y m_D v^2 \cos 2\beta}{2(m_D^2 + \Delta_s^2)} \tag{4.12}$$

The Higgs potential (neglecting s which has already been shifted to its appropriate minimum) now reads

$$\begin{aligned}
 V = & \left[|\mu|^2 + m_{H_u}^2 + \frac{g_Y^2 m_D^2 v^2 \cos 2\beta}{4(m_D^2 + \Delta_s^2)} \right] H_u^2 + \left[|\mu|^2 + m_{H_d}^2 - \frac{g_Y^2 m_D^2 v^2 \cos 2\beta}{4(m_D^2 + \Delta_s^2)} \right] H_d^2 \\
 & - (bH_uH_d + h.c.) + \frac{(g^2 + g_Y^2)}{8}(H_u^*H_u - H_d^*H_d)^2
 \end{aligned} \tag{4.13}$$

So we see the presence of the s vev amounts to a redefinition of the values of $m_{H_u}^2$ and $m_{H_d}^2$. Thus, we can perform the usual minimization and diagonalization of the MSSM Higgs fields. The new trilinear couplings induce a mixing between the s and the other Higgses, which is encoded in the mass matrix (in the $(s \ h \ H)$ basis)

$$\begin{pmatrix}
 m_D^2 + \Delta_s^2 & \frac{g_Y m_D v s_{\alpha+\beta}}{\sqrt{2}} & -\frac{g_Y m_D v c_{\alpha+\beta}}{\sqrt{2}} \\
 \frac{g_Y m_D v s_{\alpha+\beta}}{\sqrt{2}} & m_h^2 & 0 \\
 -\frac{g_Y m_D v c_{\alpha+\beta}}{\sqrt{2}} & 0 & m_H^2
 \end{pmatrix} \tag{4.14}$$

Here, as we dial up m_D^2 , which is always positive, we increase the mixing. The values of m_h^2 and m_H^2 are the usual ones from the MSSM, in particular with m_h^2 bounded at tree level to be below m_Z^2 .

Since $s_{\alpha+\beta} \equiv \sin(\alpha + \beta) > \cos(\alpha + \beta) \equiv c_{\alpha+\beta}$ generally, and $m_H^2 > m_h^2$, we can just focus on the 2×2 mixing submatrix between s and h . The mass eigenstates are related to the interaction eigenstates as given in (4.6). We are principally interested in the trilinear terms, especially the $\tilde{s}^2 \tilde{h}$ term, which allows $\tilde{h} \rightarrow 2\tilde{s}$ decays. There are two contributions

to this term, one arising from the new supersoft operator, once we have mixed the CP-even states, and one from the usual D-term trilinears, once we have mixed. The coefficients of these terms are given in the appendix.

$\lambda_h \mathbf{S} \mathbf{H}_u \mathbf{H}_d$

λ_h is a commonly studied superpotential operator, as it induces an additional Higgs quartic at small $\tan \beta$ which can raise the tree level Higgs mass [9, 8, 10]. After electroweak symmetry breaking, it generates masses for a and s of size $\lambda_h v$. With these contributions the potential is given by

$$\begin{aligned}
 V = & (|\mu|^2 + m_{H_u}^2)H_u^2 + (|\mu|^2 + m_{H_d}^2)H_d^2 - (bH_u H_d + h.c.) + \\
 & \frac{(g^2 + g_Y^2)}{8}(H_u^* H_u - H_d^* H_d)^2 + \lambda_h^2 (H_u H_d)(H_u H_d)^* \\
 & + \frac{\lambda_h^2}{2}(s^2 + a^2)(H_u^* H_u + H_d^* H_d) + \sqrt{2}\lambda_h \mu s (H_u^* H_u + H_d^* H_d) + \frac{\delta_s^2}{2}s^2 + \frac{\delta_a^2}{2}a^2
 \end{aligned}
 \tag{4.15}$$

Note the explicit μ -term for the Higgses (and the resulting trilinears), this is to be compared to the NMSSM where the vev of S gives the μ -term. However, absent any additional mass terms for S , i.e. δ_s , we can simply shift $S \rightarrow S - \sqrt{2}\mu/\lambda_h$, removing the entire μ term. In the presence of additional soft masses for s there is an s-vev of,

$$\langle s \rangle = -\frac{\sqrt{2}\lambda_h \mu v^2}{\delta_s^2 + \lambda_h^2 v^2}.
 \tag{4.16}$$

The effect of this is to replace μ by an effective μ , which we denote $\tilde{\mu}$, where

$$\tilde{\mu} = \mu \left(\frac{\delta_s^2}{\delta_s^2 + \lambda_h^2 v^2} \right).
 \tag{4.17}$$

Through the new quartic and trilinear terms there are mixings between s and the MSSM Higgses, giving a mass matrix

$$\begin{pmatrix}
 \lambda_h^2 v^2 + \delta_s^2 & -2\lambda_h v \tilde{\mu} s_{\alpha-\beta} & 2\lambda_h v \tilde{\mu} c_{\alpha-\beta} \\
 -2\lambda_h v \tilde{\mu} s_{\alpha-\beta} & m_h^2 & 0 \\
 2\lambda_h v \tilde{\mu} c_{\alpha-\beta} & 0 & m_H^2
 \end{pmatrix}
 \tag{4.18}$$

Notice, however, that in the absence of additional soft masses for s ($\delta_s^2 = 0$) the mixing vanishes, and increases as we deviate from zero. The presence of s - h mixing allows $\tilde{s} \rightarrow 2a$ decays. It is possible for $\tilde{h} \rightarrow 2\tilde{s}, 2a$ decays to occur with equal amplitude, for instance at small mixing. However, it is also possible to arrange for λ_h to give only one dominate decay of \tilde{h} . Finally, remember that the presence of λ_h naturally requires the presence of A_h , at least at a loop-suppressed level.

$A_s \mathbf{S}^3 + c.c.$

Another possible operator to add is a soft trilinear for S in the potential, $A_s \mathbf{S}^3 + c.c.$. Alone this has no effect on the Higgs, although it does give equal but opposite contributions to

s and a soft masses with an S vev and also induces $s \rightarrow 2a$ decays. However, if there is another source of $s - h$ mixing, A_s can generate $\tilde{h} \rightarrow 2\tilde{s}$ and $\tilde{h} \rightarrow 2a$ decays as well as the original $\tilde{s} \rightarrow 2a$. The effective potential with this contribution is,

$$V = (|\mu|^2 + m_{H_u}^2)H_u^2 + (|\mu|^2 + m_{H_d}^2)H_d^2 - (bH_uH_d + h.c.) + \frac{(g^2 + g_Y^2)}{8}(H_u^*H_u - H_d^*H_d)^2 + \frac{A_s}{\sqrt{2}}(s^3 - 3sa^2) + \frac{\delta_s^2}{2}s^2 + \frac{\delta_a^2}{2}a^2 \quad (4.19)$$

Since this operator does little of interest by itself it must be considered in tandem with some other operator that produces $s - h$ mixing and an s-vev. One combination of note is m_D and A_s . We can analyze the general case under the simplifying assumption, as above, that the mixing involves mainly s and h and not H , so the mass and interaction bases are related as in eq. (4.6).

The relevant trilinears are given in the appendix. As in the case of λ_h , at small mixing $\tilde{h} \rightarrow 2\tilde{s}$ and $\tilde{h} \rightarrow 2a$ have equal amplitudes. The size of the s-vev $\langle s \rangle$, generated by the mixing operator will be corrected by A_s but if $A_s \langle s \rangle^2 \ll m_s^2 \langle s \rangle$, as will often be the case, its effects can be ignored.

$A_h \mathbf{S} \mathbf{H}_u \mathbf{H}_d + c.c.$

The potential with A_h included is,

$$V = (|\mu|^2 + m_{H_u}^2)H_u^2 + (|\mu|^2 + m_{H_d}^2)H_d^2 - (bH_uH_d + h.c.) + \frac{(g^2 + g_Y^2)}{8}(H_u^*H_u - H_d^*H_d)^2 + (A_hSH_uH_d + c.c.) + \frac{\delta_s^2}{2}s^2 + \frac{\delta_a^2}{2}a^2 \quad (4.20)$$

After EWSB this generates a vev for s ,

$$\langle s \rangle = -\frac{A_h v^2}{\sqrt{2}\delta_s^2} \sin 2\beta \quad (4.21)$$

This gives a correction to the b-term for the MSSM Higgses, but still allows the usual diagonalization, leading to a mixing matrix of the form,

$$\begin{pmatrix} \delta_s^2 & A_h v c_{\alpha+\beta} & A_h v s_{\alpha+\beta} \\ A_h v c_{\alpha+\beta} & m_h^2 & 0 \\ A_h v s_{\alpha+\beta} & 0 & m_H^2 \end{pmatrix} \quad (4.22)$$

Unlike the previously discussed operators, A_h mixes the CP odd component of S with A^0 . The mixing matrix, in the basis $(a \ A^0)$, is

$$\begin{pmatrix} m_a^2 & -A_h v \\ -A_h v & m_A^2 \end{pmatrix} \quad (4.23)$$

The mass eigenstates are related to the interaction eigenstates as given in (4.27). This mixing of a with A^0 means that a can decay to 2 b-quarks or 2 τ 's if its mass is below about 10 GeV. With no additional terms in the Lagrangian, e.g. $M_Q^{-1}, m_{\text{CP}}^2 = 0$, this is the only decay path of a . The relevant trilinears are given in the appendix.

m_{CP}^2 $s a$ — CP Mixing Mass

The operator m_{CP}^2 is a CP mixing mass which mixes the would-be pseudoscalar a with the would-be scalar s . This operator can be important in allowing decays of the a when M_Q^{-1} and A_h are zero and can be particularly important when decays $h \rightarrow 2s$ would dominate the Higgs decay, but are kinematically forbidden. In this case, mixing due to m_{CP}^2 can make $h \rightarrow sa$ the dominant decay mode.

We should point out that this operator does not, *by itself*, violate CP. As mentioned earlier, without any other interactions, there is nothing to prevent us from assigning CP-even transformation properties to both a and s . This remains true with the inclusion of A_s or the supersoft operator m_D in the potential. Only when a couples to fermions or gauge bosons as a pseudoscalar, or mixes with the A^0 is a CP-odd property forced upon it. The scenarios which we will study will not have such a mixing and hence no actual CP violation is introduced into the theory, making such a term safe from edm searches, for example.

One requires either m_D or λ_h to induce Higgs decays (through direct couplings or mixings). In the case that λ_h is present, $h \rightarrow 2a$ is already significant, so $h \rightarrow as$ typically cannot compete. However, in the m_D case, where a has no couplings, such a scenario is viable. We thus consider the mass matrix (in the $(a s h)$ basis)

$$\begin{pmatrix} \delta_a^2 & m_{\text{CP}}^2 & 0 \\ m_{\text{CP}}^2 & m_D^2 + \delta_s^2 & \frac{g_Y m_D v s_{\alpha+\beta}}{\sqrt{2}} \\ 0 & \frac{g_Y m_D v s_{\alpha+\beta}}{\sqrt{2}} & m_h^2 \end{pmatrix} \quad (4.24)$$

Because the mixing will allow the a to be produced through a -strahlung from the Z, LEP limits become very severe. Ultimately, this implies that m_{CP}^2 must be somewhat small, and for our purposes here, we will treat it as a perturbation. Ultimately, it will be necessary to calculate mixings precisely, but for estimates and intuition, it is easiest to study in the perturbative limit. We can perform a rotation in the sh sector by an angle θ , which may be large. This leaves the following matrix

$$\begin{pmatrix} m_a^2 & m_{\text{CP}}^2 \cos \theta & m_{\text{CP}}^2 \sin \theta \\ m_{\text{CP}}^2 \cos \theta & m_{\tilde{s}}^2 & 0 \\ m_{\text{CP}}^2 \sin \theta & 0 & m_{\tilde{h}}^2 \end{pmatrix} \quad (4.25)$$

The mass matrix can then be diagonalized by a series of rotations

$$U = R(\theta_{sh})R(\delta_{as})R(\delta_{ah}) \quad (4.26)$$

where $\theta_{sh} = \theta$ diagonalizes the sh mass matrix, and the $R(\delta_i)$ are perturbative rotations where $\sin \delta_{as} = m_{\text{CP}}^2 \cos \theta / (m_{\tilde{s}}^2 - m_a^2)$ and $\sin \delta_{ah} = m_{\text{CP}}^2 \sin \theta / (m_{\tilde{h}}^2 - m_a^2)$, where $m_{\tilde{s}, \tilde{h}}^2$ are the $\tilde{s}\tilde{s}$ and $\tilde{h}\tilde{h}$ entries after the θ rotation.

The relevant mixing angle which enters into \tilde{a} -strahlung is

$$\begin{aligned} \sin \theta_{ah} &= \sin \theta \cos \delta_{ah} \sin \delta_{as} - \cos \theta \sin \delta_{ah} \\ &\simeq m_{\text{CP}}^2 \sin \theta \cos \theta \left((m_{\tilde{s}}^2 - m_a^2)^{-1} - (m_{\tilde{h}}^2 - m_a^2)^{-1} \right). \end{aligned} \quad (4.27)$$

M_Q^{-1} — **Fermiophobic Decay Operator**

If the singlet superfield couples to new vector matter, there are loop induced decays of the scalars into gauge bosons. The presence of the new fields corrects the beta function of the SM gauge groups, with a superpotential coupling

$$\lambda_Q S Q \bar{Q} + M_Q Q Q \bar{Q} \tag{4.28}$$

the gauge kinetic term becomes

$$\int d^2\theta \left(\frac{1}{2g^2} + \frac{b_i}{16\pi^2} \log(m_Q + \lambda_Q S) \right) W_\alpha W^\alpha \simeq \left(\frac{1}{2g^2} + \frac{b_i}{16\pi^2} M_Q^{-1} S \right) W_\alpha W^\alpha \tag{4.29}$$

where W_α is the gauge field strength, and $M_Q^{-1} = \lambda m_Q^{-1}$. This results in a coupling between s and two gauge fields, as well as a and two gauge fields which explains why we refer to this as a fermiophobic decay operator. Because s generally mixes with the Higgs, it is difficult for a loop suppressed decay to compete. However, a , which mixes with the often heavy A^0 , and often through the loop suppressed operator A_h , can have its dominant decay mode through this operator. Expanding out the expression above, we find a term in the Lagrangian

$$\mathcal{L} \supset c(a F^a \tilde{F}^a) \equiv c(a \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a)/2 \tag{4.30}$$

This induces a decays into photons or gluons, with the decay width

$$\Gamma_c = \frac{c^2}{4\pi} m_a^3 N_D \tag{4.31}$$

where N_D is the number of “colors” in the final state (i.e. 1 for photons and 8 for gluons).

With the convention above,

$$c = \frac{\sqrt{2} M_Q^{-1}}{64\pi^2} b_i g_i^2 = \frac{\sqrt{2} M_Q^{-1}}{16\pi} b_i \alpha_i \tag{4.32}$$

where i labels the gauge group and b_i is its beta function where $\frac{dg_i}{d \log \mu} = b_i \frac{g_i^3}{16\pi^2}$. In the specific case of a $5 + \bar{5}$ and higgsinos

	D, D^c	L, L^c	\tilde{H}_u, \tilde{H}_d
b_3	1	0	0
b_Q	2/3	2	4/3

(4.33)

We can compare the decays into gluons and those into b quarks through mixing,

$$\Gamma_{\text{glue}}/\Gamma_{b\bar{b}} = \frac{(N_c^2 - 1)c^2 m_a^3 / (4\pi)}{(m_b/v)^2 N_c (\sin \phi \tan \beta)^2 \sqrt{1 - 4m_b^2/m_a^2} (m_a/16\pi)} \tag{4.34}$$

which for the particular case of a D, D^c , becomes

$$\Gamma_{\text{glue}}/\Gamma_{b\bar{b}} = \frac{1}{4\pi^2} \frac{\alpha_3^2}{N_c (\sin \phi \tan \beta)^2 \sqrt{1 - 4m_b^2/m_a^2}} \left(\frac{m_a v}{m_b M_Q} \right)^2 \tag{4.35}$$

For decays to glue to dominate over b's, one requires small mixing angles in the CP-odd sector, which can be achieved simply with loop suppressed A_h , or a heavy A^0 . Alternatively, one can have lighter D, D^c squarks, for example, with the values $\alpha_3 = .12, m_a = 40\text{GeV}, M_Q = 330\text{GeV}$ one requires $(\sin \phi \tan \beta) \lesssim .05$ to get comparable rates.

When M_Q^{-1} induced decays for \tilde{a} dominate, we can get an estimate on the branching ratios into gluons and photons in the case of S coupling to a $SU(5)$ complete multiplet. For this, the branching ratios are $(2g, 2\gamma) = (.99, 3.8 \times 10^{-3})$. If the Higgs dominantly decays into $2\tilde{a}$, this gives branching ratios $\tilde{h} \rightarrow 2\tilde{a} \rightarrow (4g, 2g, 2\gamma, 4\gamma)$ of $(.99, 7.6 \times 10^{-3}, 1.5 \times 10^{-5})$. The channels with photons may be enhanced by including λ_h which only induces decays into photons, via Higgsino loops. As a final comment on this operator, we point out that if this is the only allowed \tilde{a} decay, as M_Q increases, \tilde{a} will first decay with a visible displaced vertex and eventually will decay outside of the detector. Both of these possibilities are highly constrained by LEP searches if the Higgs can be produced, ruling out such a Higgs that cascade decays into two such \tilde{a} scalars.⁶ Therefore, we only consider values of this operator where \tilde{a} decays promptly.

Other Operators

Note that we ignore here the effects of a μ -term for the S field, and a superpotential S^3 term. Since we are mostly concerned with the scalar sector of the theory, and aren't concerning ourselves with the cosmological implications at this time, the mass of the singlino is irrelevant for our purposes. The only effect of μ_S is then to generate an $S^* H_u H_d$ term in the potential in a cross term with λ_h . This has the same phenomenological effect as A_h , so including μ_S should not change the basic outcome of our study. Including S^3 in the superpotential can induce invisible decays of the Higgs to a singlino, but these are highly constrained and not interesting for our purposes. It can also induce a term $H_u H_d S^{*2}$ in the potential, but this acts simply like A_h in the presence of an s -vev, and like λ_h in the presence of an h -vev in allowing $h \rightarrow 2a, 2s$ decays. Hence, we do not consider either operator here.

5. Scenarios

Although we have expanded the MSSM only by a single superfield, there is a remarkably large number of new scenarios of Higgs decays which arise, with large variations in the number of particles in the final state, as well as types of particles in the final state.

We can group the scenarios, in increasing order of complexity, into three categories:

- The Higgs is mixed heavy. In the presence of large mixing, the Higgs mass can be increased significantly, in spite of small radiative corrections. In this scenario, the Higgs is at least 114 GeV in the case of SM-like decays, or 106 GeV in the case of cascade decays to 4 b quarks.

⁶Note added: It has recently been emphasized that similar Higgs decays with highly displaced vertices could well be visible at Tevatron and LHC, enhancing its detection prospects [40].

- The Higgs decay is dominated by a single stage decay. Here $h \rightarrow 2a \rightarrow 2X$ or $h \rightarrow 2s \rightarrow 2X$, where X is composed of a pair of standard model fields. As above, if $X = 2b$, one requires $m_h \gtrsim 106\text{GeV}$, however, if $X = 2\tau$ or $X = 2g$, the Higgs can be considerably lighter.
- The Higgs decay is dominated by a two-stage cascade. That is $h \rightarrow 2s \rightarrow 4a \rightarrow 4X$ or $h \rightarrow as \rightarrow 3a \rightarrow 3X$. Such two stage cascades generally do not occur if one restricts oneself to the scenario of the NMSSM. Here $X = 2\tau, 2b, 2g$, resulting in 6 and 8 particle final states, which have not been constrained by LEP analyses other than the model independent ones.

Additionally, we can split the scenarios further into two cases: “large-mixing” and “small-mixing”. We define these as follows: the small mixing case arises when the light CP-even singlet has a sufficiently small Higgs component that s-strahlung limits do not constrain it, even with conventional decays. In the large-mixing case, the singlet has sufficient Higgs component that non-standard decays are needed to evade limits.

With large mixing case, there are three real scalars that we are concerned with: h , s , and a . These are defined to be the field which couples to the Z in Higgs-strahlung, the field which mixes with the Higgs, and the field which does not mix with the Higgs, respectively. In many cases s can be thought of as the scalar and a as the pseudoscalar. The mass eigenstates are \tilde{h} , \tilde{s} and \tilde{a} , which are the fields mostly made of the untilded field of the same character, and usually the heaviest, intermediate, and lightest mass eigenstates, respectively. As a note of caution, we will often refer to the $\tilde{h}, \tilde{s}, \tilde{a}$ still as the Higgs, CP-even singlet and CP-odd singlet and have tried to ensure that the meaning is clear given the context.

First, we will discuss the necessary operators in all scenarios and then we will proceed to study the three basic scenarios, including the existing limits and model building possibilities.

Model Building: Necessary Operators in All Scenarios

The basic requirements on any viable scenario are threefold, and can be satisfied with various combinations of operators. The requirements [operators satisfying the requirements] are:

- \tilde{a} must decay [$A_h, m_{\text{CP}}^2, M_Q^{-1}$]: This is not absolutely true, for instance if \tilde{a} is never produced, but almost always is.
- \tilde{h} must decay into $2\tilde{s}$ or $2\tilde{a}$ or $\tilde{a}\tilde{s}$ [m_D, λ_h, A_s (+ mixing), A_h]: For the mass range we are interested in, these non-standard decays for \tilde{h} must exist.
- \tilde{s} must decay to $2\tilde{a}$ [λ_h (+ mixing), A_s, A_h (+ mixing), m_{CP}^2]: When \tilde{s} is light (i.e. $\lesssim 80\text{ GeV}$) and in “large-mixing”, constraints from SM decays are usually quite stringent. Conceivably, \tilde{s} could decay directly primarily to τ 's if the mixing is small enough, but in general, the cascade decays for \tilde{s} are necessary to evade LEP limits, unless it is heavier than \tilde{h} .

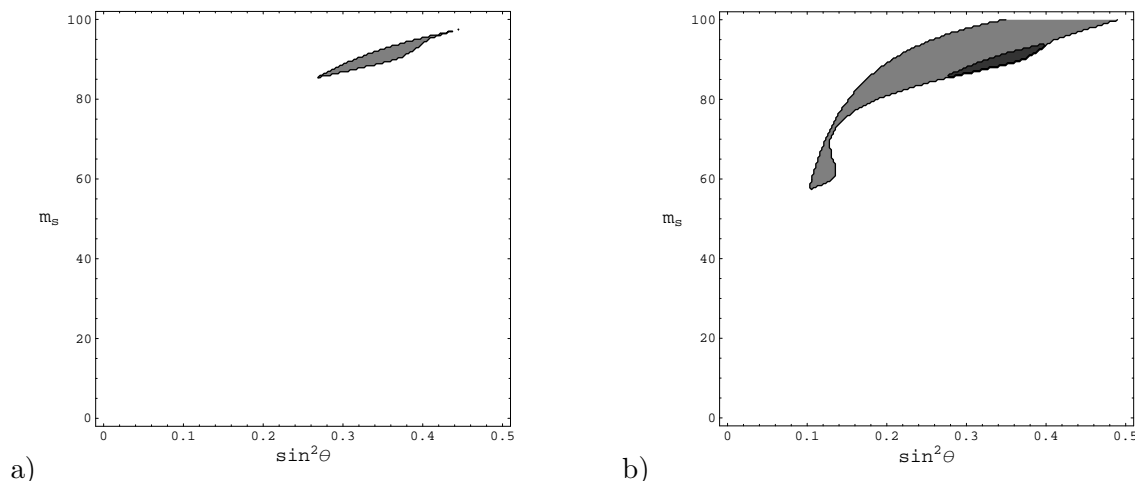


Figure 1: The allowed regions in $\sin^2 \theta$ - $m_{\tilde{s}}$ space, where $m_{\tilde{s}}$ is the mass of a light singlet mixing with the Higgs an amount $\sin \theta$. The plots are shown with $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 270$ GeV (a) and $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 300$ GeV (b). The shaded regions (combined dark and light) are allowed from the combined LEP limits, while the dark shaded regions additionally have $m_{\tilde{h}} > 110$ GeV.

5.1 Higgs Decays Through a Single Stage Cascade Into b Quarks

As we have already described, the only means in the MSSM to push up the mass of the Higgs above LEP limits is to introduce very heavy top squarks, which then introduce tunings into the theory. However, as described in section 4, if the Higgs mixes with a lighter field s , the mass of the field \tilde{h} which is produced strongly through \tilde{h} -strahlung from a Z can have its mass pushed up, simply by mass mixing, without introducing unnaturally heavy top squarks. However, this mixing allows the lighter field to be produced weakly through \tilde{s} -strahlung, trading off the heavier \tilde{h} for stronger limits on the lighter \tilde{s} .

Now that \tilde{s} contains a Higgs component, we must consider LEP limits on it. With standard model decays for \tilde{s} , the limits are typically $\sin^2 \theta \lesssim 0.03$. The large mixing necessary to achieve changes in the \tilde{h} mass requires that \tilde{s} decays dominantly through cascades, in this case, one typically requires $\sin^2 \theta \lesssim 0.2$ if the cascades end in b quarks for lighter \tilde{s} and $\sin^2 \theta \lesssim 0.4$ for heavier \tilde{s} . These, in general, also imply that the dominant decays of the Higgs are non-standard.

If these decays are $\tilde{h} \rightarrow 4\tau$ or $\tilde{h} \rightarrow 4g$, then the Higgs can be considerably lighter than 115 GeV, and it is essentially unnecessary to push the Higgs mass up. These scenarios will be discussed in section 5.2.

We show the allowed regions (dark and light shaded) with \tilde{h} decays to four b jets in figures 1a and 1b. In these plots, we assume a value for $m_a > 12$ GeV (where kinematics no longer strongly favor decays to τ 's) where constraints are strongest. We show the regions which are allowed when accounting for constraints on $\tilde{s} \rightarrow 4b$ decays (light and dark shaded regions), assuming a typical $BR_{a \rightarrow 2b} \simeq 0.87$. For comparison, we also plot the region where $m_{\tilde{h}} > 110$ GeV, where $h \rightarrow 4b$ is essentially unconstrained (dark shaded regions).

These plots have assumed a 100% branching ratio for $h \rightarrow 2a$, so the only applicable limits are the $h \rightarrow 4b$ exclusions given in the LEP-wide analysis [18, 19]. However, it is an important question what the LEP constraints are on a scenario with both $h \rightarrow 2b$ and $h \rightarrow 4b$ rates. In a recent paper on these decays in the NMSSM [41], the authors only required that the $h \rightarrow 2b(4b)$ rate be consistent with the LEP $h \rightarrow 2b(4b)$ exclusions, thus they assumed that these analyses are independent of each other. However, upon closer inspection of the details of the analyses, it appears such an approach may be too generous.

In the higher mass ($m_{\tilde{h}} > 90\text{GeV}$) range, the combined LEP analyses are dominated by OPAL [33] and DELPHI [34]. A close reading of these papers suggests that the $2b$ and $4b$ decays of the Higgs are generally reconstructed together. For instance, in studying $e^+e^- \rightarrow Zh$, in the $Z \rightarrow \nu\nu$ analysis by OPAL, the neural nets trained to capture the $4b$ decays of the Higgs are also reasonably efficient in capturing the $2b$ decays. In the all jet ($Z \rightarrow jj$) analysis, OPAL uses the same analysis procedure for both $2b$ and $4b$, forcing the six jet event into a four jet topology via the DURHAM algorithm, hence this analysis efficiently reconstructs both types of decays. As for DELPHI, they clearly state that the analyses are not independent of each other [42], and both decays are reconstructed via the same analysis procedure.

Beyond efficiencies, there are still differences between $2b$ and $4b$ events. For example, the distribution of discriminating variables will be broader in $4b$ decays (e.g. the reconstructed Higgs mass), making it difficult to know how to constrain the scenario when there are significant levels of both $2b$ and $4b$ decays. If $2b$ and $4b$ signals were indistinguishable, the correct limit would be $\frac{\xi_{4b}^2}{\xi_{4b,bd}^2} + \frac{\xi_{2b}^2}{\xi_{2b,bd}^2} < 1$, where ξ_{bd}^2 is the experimental bound of the individual analysis. As an attempt to combine these limits, when there are $O(1)$ rates for $2b$ and $4b$ decays, in addition to applying the individual limits, we will also require that

$$\xi_{2b+4b}^2 \equiv \frac{\xi_{4b}^2}{\xi_{4b,bd}^2} + \frac{\xi_{2b}^2}{\xi_{2b,bd}^2} < \sqrt{2} \tag{5.1}$$

This additional requirement, on the effective ξ_{2b+4b}^2 , accounts for the redundancy of the analyses, and which should forbid situations where the naive combination of analyses is significantly excluded. Thus, we find it to be a good compromise between the assumptions of complete independence/interdependence ($\xi_{2b+4b}^2 \leq 2, 1$ respectively) of the separate analyses. Note that one can also interpret this as only requiring a 99.5% CL limit, if the two decays give indistinguishable events. More rigorously, a combined analysis should be done to find the proper constraints.

For our purposes here, where we take the branching ratio $h \rightarrow 2a$ to be one, the results are relatively simple. Even with relatively light stops (270 GeV), the small suppressions of the Higgs couplings due to mixing are sufficient to allow such a Higgs to have been undetected. However, at this mass, this is achieved by strongly mixing and pushing the physical Higgs above 110 GeV(out of the LEP constraints). At $m_{\tilde{t}} = 300$ GeV, a region opens up below 110 GeV, but the overall allowed space is still quite narrow. Moving to higher Higgs mass (roughly 325 GeV stops) the model independent parameter space opens up significantly (about twice as large as at 300 GeV). However, even with 300 GeV stops,

the tuning of m_Z is already expected to be $O(15\%)$, which begins to reintroduce fine tuning from another direction.

Note that in all of these cases we are considering \tilde{s} to be lighter than \tilde{h} . The reason for this is simple: although mixing the Higgs with a heavier singlet can suppress couplings of the \tilde{h} , if it is heavier than the Higgs, the effect is to push the mass of \tilde{h} down, aggravating naturalness issues. Thus, we should view a light \tilde{s} as a natural consequence of this model with $m_{\tilde{h}} \sim 110$ GeV. Such a scenario is somewhat distinct from previously discussed scenarios [43, 41] because of the presence of the light \tilde{s} which can be easily produced at rates comparable to that of the SM (in general, roughly a factor of 10 smaller).

Let us further note that we have been discussing the model independent tuning. Whether or not one can achieve these mixings in a given model without, e.g., tuning to prevent a tachyonic \tilde{s} is a separate, often more stringent constraint.

As a final comment, we note that these limits are based on the best available limits of $h \rightarrow 2a \rightarrow 4b$, which are still preliminary. Final limits may further constrain this scenario.

Model Building: Single Stage Cascades into b Quarks

It is straightforward to construct models in which the Higgs is mixed strongly with a lighter scalar. Because of this mixing, if the Higgs decays $\tilde{h} \rightarrow 2a$, then generally \tilde{s} decays to $2a$ as well. Here we will discuss the model building and tunings associated with large mixings, and situations where $\tilde{h} \rightarrow 2a \rightarrow 4b$. Situations with $\tilde{h} \rightarrow 2a \rightarrow 4g, 4\tau$, where Higgs mass may be below 110 GeV, will be deferred to subsequent discussions.

There are three operators which can induce significant mixing with the Higgs: m_D , λ_h , and A_h . A_h principally mixes s with H^0 rather than h because $\cos(\alpha + \beta)$ is small, so we focus on the other two.

m_D is unique in that while it induces mixing, it also adds a diagonal mass for s , so that a tachyon never appears. As a consequence, with this operator, it is simple to get large mixing without having to tune masses to a high degree. In figures 2 we see that one can easily achieve large Higgs masses and large mixings over broad ranges of the parameter space.

λ_h is somewhat more challenging, because we require a sufficiently large μ -term from experiment. From searches for the chargino, we typically require $\mu > 100$ GeV [44]. In figure 3 we consider this scenario for $\lambda = 0.25$. Here we see that it is very challenging to have sufficiently large μ while keeping the light state dominantly s (i.e., $\sin^2 \theta < 0.5$). One can tune this scenario to achieve this, but generally, it is most natural to have $\sin^2 \theta \sim 1$ and the s decoupled, and $m_h > 110$ GeV (as is required with no singlet component). Naturalness here is not considerably improved from the MSSM, unfortunately.

Since λ_h is in general more tuned, we list a benchmark with m_D below (masses in GeV and in the decoupling limit which we define as $m_A \gg m_Z$ and large $\tan \beta \gtrsim 5$)

$m_{\tilde{t}}$	δ_s^2	δ_a^2	m_D	A_s	$ \sin^2 \theta $	$m_{\tilde{h}}$	$m_{\tilde{s}}$	$m_{\tilde{a}}$	$B(\tilde{h} \rightarrow 2\tilde{a})$	$B(\tilde{s} \rightarrow 2\tilde{a})$	tuning
325	45 ²	57 ²	43	-11	0.1	109	73.8	32.6	0.86	.99	3%

(5.2)

Here “tuning” is defined as the tuning in m_D necessary to satisfy the necessary requirements. The maximum value of m_D arises from the limit on $\tilde{s} \rightarrow 2a$ decays and the

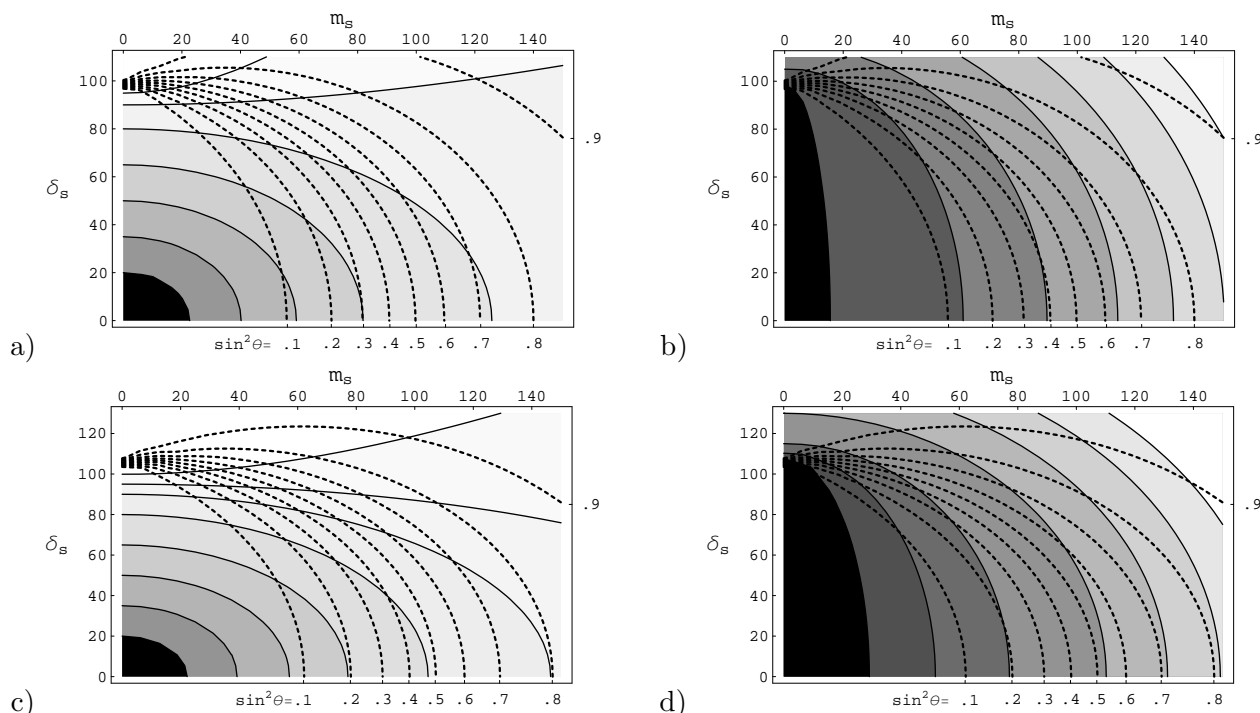


Figure 2: In these plots, the contours are of ranges of \tilde{s} mass (a,c) or \tilde{h} mass (b,d). The x-axis specifies the value of m_s arising from the m_D operator. The y-axis is all other contributions to the mass squared term for s . The values of $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ are 250 GeV for a,b and 350 GeV for c,d. The dashed contours are lines of constant $\sin^2 \theta$ as labeled. The contours (darkest to lightest) are for masses less than a) 20, 35, 50, 65, 80, 90, 95; b) 100, 105, 115, 130, 145, 160, 175; c) 20, 35, 50, 65, 80, 90, 95, 100; d) 108, 110, 115, 130, 145, 160, 175.

minimum value arises from the lower limit on $m_{\tilde{h}}$. The tuning of a parameter x which can lie between two values a and b , with $b > a$ will be defined to be the smaller of 100% or $(b - a)/(b + a)$. Qualitatively, this is the average fractional change in a parameter which is allowed consistent with phenomenology and constraints. In future benchmarks, it will generally be defined as the largest tuning in δ_s^2 or δ_a^2 to achieve the proper spectrum of \tilde{s}, \tilde{a} masses. This tuning is usually related to the lighter of the two states. In these scenarios, constraints on ξ_{2b+4b}^2 (see eq. 5.1) are very stringent, so this region may not be allowed by the full analysis.

5.2 Higgs Decays Through a Single Stage Cascade (without b's)

Single stage cascade decays of the Higgs are familiar within supersymmetric theories. Within the CP violating MSSM such decays can easily occur [45–47]. Within the NMSSM, the decays $h \rightarrow 2a \rightarrow 4\gamma$ are well studied [36, 48], while it has been argued, prior to the combined LEP analyses, that $h \rightarrow 2a \rightarrow 4b$ allowed considerable reduction in the fine tuning of m_Z within the NMSSM [17]. More recently, it has been argued that the combined LEP limits favor a $h \rightarrow 2a \rightarrow 4b$ decaying Higgs at approximately $m_h \sim 105$ GeV [41], which we argued in section 5.1, had applied constraints that may have been too generous.

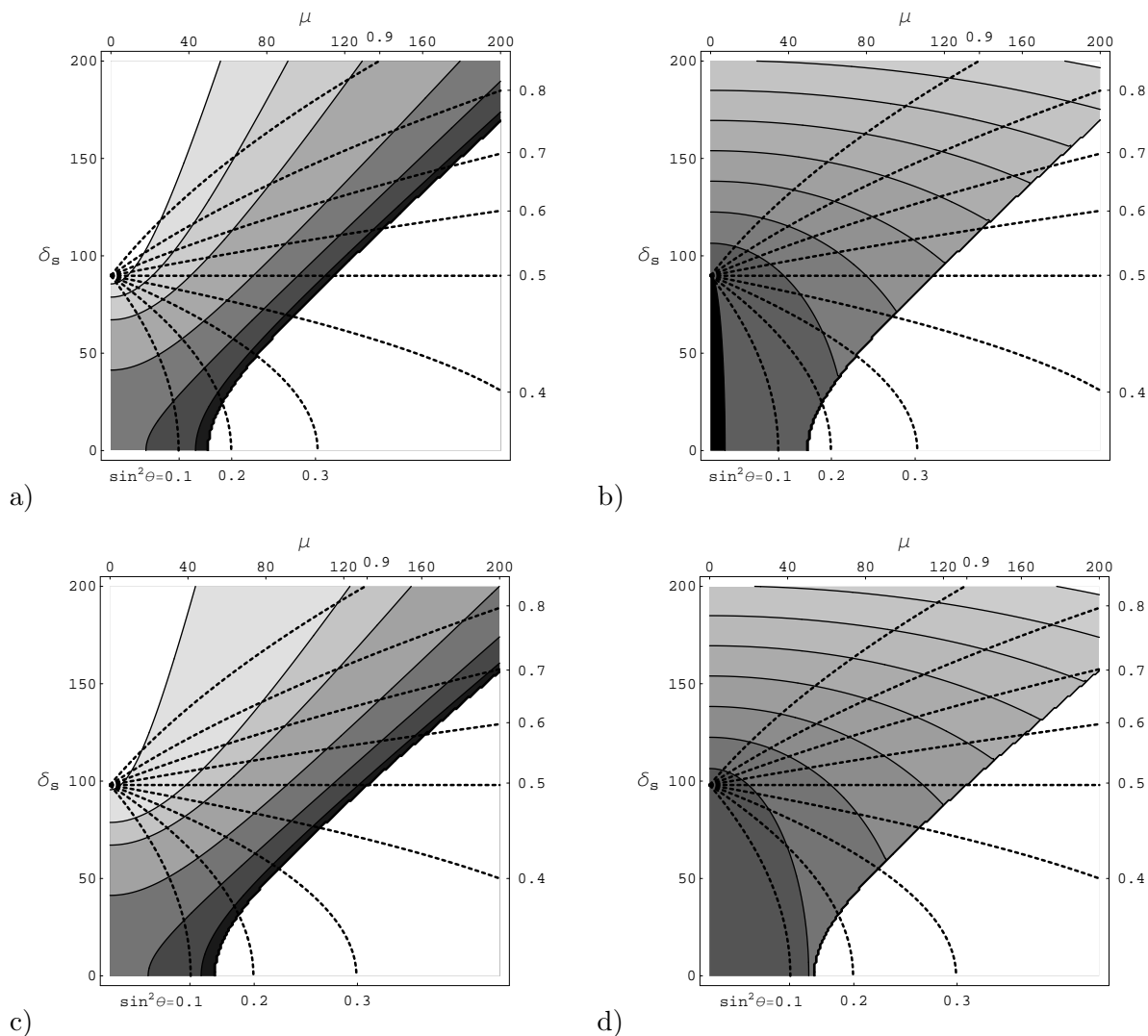


Figure 3: In these plots, the contours are of ranges of \tilde{s} mass (a,c) or \tilde{h} mass (b,d). Here the λ_h operator is taken to be $\lambda = 0.25$. The x-axis is the value of the effective μ parameter (once s has acquired a vev). The y-axis are other soft contributions to the mass squared of s , in addition to the $\lambda^2 v^2$ term arising from λ_h . The values of $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ are 250 GeV for a,b and 350 GeV for c,d. The dashed contours are lines of constant $\sin^2 \theta$ as labeled. The contours (darkest to lightest) are for masses less than a) 20, 40, 60, 80, 90, 96; b) 100, 115, 130, 145, 160, 175, 190, 205, 220; c) 20, 40, 60, 80, 90, 105; d) 105, 115, 130, 145, 160, 175, 190, 205, 220. The white region in the lower right corner is where \tilde{s} is tachyonic.

$h \rightarrow 2a \rightarrow 4\tau$ has also been considered [48, 43]. Here we study these, and an additional possibility, namely, $\tilde{h} \rightarrow 2\tilde{a} \rightarrow 4g$.

$\tilde{\mathbf{h}} \rightarrow 2\tilde{\mathbf{s}}, 2\tilde{\mathbf{a}} \rightarrow 4\tau$: Phenomenologically, it makes little difference whether the intermediate state here is $2\tilde{\mathbf{s}}$ or $2\tilde{\mathbf{a}}$, except if it is $2\tilde{\mathbf{s}}$, the $\tilde{\mathbf{a}}$ can be completely decoupled, while if it is $2\tilde{\mathbf{a}}$, the $\tilde{\mathbf{s}}$ could mix considerably with the Higgs, giving large corrections to the Higgs couplings.

In general, models of this sort are highly tuned, because the mass of the intermediate scalar must be $\lesssim 10$ GeV in order for decays to τ 's to dominate over decays to b quarks. As a consequence, they generally do not lead to a significantly less tuned theory than the MSSM, although they offer interesting and distinct phenomenology.⁷ Generically, one expects a scalar field with mass $\lesssim 10$ GeV, as well as accompanying decays $\tilde{h} \rightarrow 2\tilde{s}, 2\tilde{a} \rightarrow 2\tau 2c$ and $\tilde{h} \rightarrow 2\tilde{s}, 2\tilde{a} \rightarrow 4c$. There may also be some interesting B physics in this mass range [50].

The only existing analyses on this scenario [22, 18, 19] cut off at $m_{\tilde{h}} > 86$ GeV, for theoretical reasons related to the MSSM which do not exist in theories with additional singlets. The extension of this analysis is important to understanding the true limits on such a scenario.

$\tilde{h} \rightarrow 2\tilde{a} \rightarrow 4g$: One of the most interesting possible signatures results in the 4 glue final state. Such a final state is difficult to see at LEP and has not been constrained by any existing analysis, save the model independent analysis [22], allowing such a Higgs, in principle, as light as 82 GeV. The extent to which we believe other analyses (e.g., the two-jet flavor independent decays) can be interpreted to constrain this scenario will be discussed in the benchmark summary.

Models with this final state can be easily constructed with essentially no fine-tuning, and squarks as light as the direct experimental limit. Although a search for four jets at a hadron collider seems impossible, the same processes which generate the four jet final state can also generate $\tilde{h} \rightarrow 2\tilde{a} \rightarrow 2g2\gamma$, which should occur with a rate down by roughly $\alpha_{EM}^2/\alpha_s^2 \sim 10^{-2}$, see M_Q^{-1} subsection of section 4. This rate can be increased if couplings of the a to Higgsinos exist as well. A search for the two photons at the LHC should be feasible [36], and there may be background reduction by requiring two jets which reconstruct to the invariant mass of the photon pair (however, this is unlikely at a hadron collider like the LHC).

Model Building: $\tilde{h} \rightarrow 2\tilde{a} \rightarrow 4\tau, 4g$ via λ_h, M_Q^{-1}

λ_h allows decays to both \tilde{s} and \tilde{a} . However, λ_h generally mixes the CP even s with h , giving strong constraints from \tilde{s} -strahlung. Consequently, with λ_h , the most natural models are those with $\tilde{h} \rightarrow 2\tilde{a} \rightarrow X$. In order to evade constraints on \tilde{s} -strahlung, it is most straightforward to simply raise the mass of \tilde{s} through the addition of δ_s^2 . In doing so, however, one generally realizes the large mixing scenario described in section 5.1, requiring that $m_{\tilde{s}} > 2m_{\tilde{a}}$ in order to evade constraints. One can raise the mass of \tilde{s} much larger than the other relevant scales, as well. In fact, this is quite reasonable, for instance if S appears as the N=2 completion of a 5-D $U(1)$ gauge multiplet. 5-D gauge invariance protects the mass of a , but not s against tree level SUSY breaking terms [51, 52]. Thus, for our purposes in this section, we will decouple s from the theory.

For $\tilde{h} \rightarrow 2\tilde{a}$ to sufficiently dominate over $\tilde{h} \rightarrow 2b$, a suitably large λ_h is necessary, and by the discussion in section 4, we see that the requirement is $\lambda > 0.2$. At tree level, this results in mass squared of $(35 \text{ GeV})^2$ for \tilde{a} .

⁷In our scenarios \tilde{h} decays into $2\tilde{a}$ are correlated with the size of the \tilde{a} mass which leads to tuning for \tilde{a} to be lighter than two b's. If the decays are mediated by derivative couplings, e.g. a Peccei-Quinn axion, then this light \tilde{a} can be reasonably natural (see for e.g. [49]).

Let us now consider the subsequent decays of \tilde{a} . If we include the a nonzero M_Q^{-1} , the possibility of $\tilde{a} \rightarrow 2g$ opens up. However, the presence of λ_h requires the presence of A_h , which allows $\tilde{a} \rightarrow 2b$ as well. This operator need not be large, being generated at the loop level. At this size

$$A_h \sim \frac{\lambda g^2}{16\pi^2} m_{\text{susy}} \sim .2 \text{ GeV} \quad (5.3)$$

it would be expected to yield a mixing of a and A^0 at the 10^{-3} level or smaller, which is easily small enough for the decays to glue to dominate. In comparison, in the NMSSM it is difficult to have M_Q^{-1} induced decays dominate because the cross term in $|\kappa S^2 + \lambda H_u H_d|^2$ acts like A_h with an S vev and thus fermion decays tend to dominate in this scenario.

The benchmark point then has (taking the decoupling limit)

$m_{\tilde{t}}$	δ_s^2	δ_a^2	λ_h	A_h	M_Q^{-1}	$ \sin^2 \theta $	$m_{\tilde{h}}$	$m_{\tilde{s}}$	$m_{\tilde{a}}$	$B(\tilde{h} \rightarrow 2\tilde{a})$	tuning
200	500^2	-35^2	0.3	0.3	300^{-1}	10^{-3}	92.1	503	38.7	.92	60%
		(-51^2)			(0)				(11.1)	(0.96)	(3%)

(5.4)

Here we have taken $\mu = 100$ GeV, there is no reduction in the effective μ -term since δ_s is large. The inclusion of a nonzero M_Q^{-1} as described in subsection M_Q^{-1} of section 4 generates the decays to two gluons. Due to the kinematics (heavy \tilde{h}, \tilde{a}), there are no constraints on these 4 jet decays.

If $M_Q^{-1} = 0$, one expects decays to standard model fermions to dominate. For decays of taus to dominate requires $m_{\tilde{a}}$ to be less than the $b\bar{b}$ threshold. This requires us to tune δ_a^2 to cancel the tree-level mass of $m_a^2 \sim (40\text{GeV})^2$. For instance, the above point with δ_a^2 of $-(51\text{GeV})^2$ achieves the proper spectrum with a 3% tuning. The necessary changes to the parameters and the corresponding changes in the spectrum are shown in parentheses in the table in the second line, whereas things that are common to the first are not repeated.

Model Building: $\tilde{h}, \tilde{s} \rightarrow 2\tilde{a} \rightarrow 4g, 4\tau$ via m_D, A_s, M_Q^{-1}

Cascade decays of the CP-even states into $2\tilde{a}$ can easily dominate with the inclusion of m_D and A_s . The operator A_s induces a large trilinear sa^2 while m_D induces $O(1)$ mixing between s and h . Thus if the $2\tilde{a}$ channel is kinematically accessible, \tilde{h}, \tilde{s} will dominantly decay into it. In addition, there is no mixing in the CP-odd sector, thus M_Q^{-1} provides the only decay channel for \tilde{a} ; hence the final state of the cascade decay is dominantly $4g$ and, at a subdominant but potentially interesting rate, $2g2\gamma$ and 4γ . Notice that in this scenario the vev of s has no relation to the μ parameter and the minima are only local minima due to the A_s operator.

As a benchmark point, we take (in the decoupling limit)

$m_{\tilde{t}}$	δ_s^2	δ_a^2	m_D	A_s	$ \sin^2 \theta $	$m_{\tilde{h}}$	$m_{\tilde{s}}$	$m_{\tilde{a}}$	$B(\tilde{h} \rightarrow 2\tilde{a})$	$B(\tilde{s} \rightarrow 2\tilde{a})$	tuning
175	80^2	0	30	8	.22	94.9	76.2	28.3	.92	.99	100%
		(-27^2)						(8.37)	(.93)		(10%)

(5.5)

Some comments on this benchmark are in order. Looking at the first line, it is a large mixing point, where \tilde{s}, \tilde{h} are similar in mass and the main constraints are on the \tilde{h} decays.

As mentioned above, with no A_h operator, there is no mixing in the CP-odd sector so \tilde{a} can only decay into gauge bosons through M_Q^{-1} . The constraint from the dominant $4g$ final state only requires $m_{\tilde{a}} \gtrsim 5$ GeV [35], so these scenarios are viable and quite natural.

As a variation on the phenomenology, τ final states can also be considered. For the 4τ state to compete, one could include a small A_h operator and tune the \tilde{a} mass below $b\bar{b}$ threshold, at the expense of increased tuning. As before the necessary changes in parameters are given in parentheses in the second line. The constraints on cascades into τ now apply, which generally require that $m_{\tilde{a}} \gtrsim 5$ GeV if $B(\tilde{a} \rightarrow 2\tau) \sim 1$. However, these points are on the edge of a constraint contour, for instance when $m_{\tilde{s}}$ is between 45-76 GeV, one instead requires $m_{\tilde{a}} \gtrsim 7$ GeV, severely limiting the allowed parameter space [35, 18, 19]. In summary, these benchmark points suggest that cascades into τ 's seem to be disfavored by both tuning considerations and available parameter space, but still remain an interesting phenomenological possibility.

5.3 Higgs Decays Through a Two Stage Cascade

A very interesting phenomenological possibility is that the Higgs cascade decay proceeds through two intermediate scalars. This is a very reasonable possibility in supersymmetric theories, because the new singlet actually comes with both the scalar and pseudoscalar, so $\tilde{h} \rightarrow 2\tilde{s} \rightarrow 4\tilde{a} \rightarrow 4X$ is possible or with CP mixing, $\tilde{h} \rightarrow \tilde{s}\tilde{a} \rightarrow 3\tilde{a} \rightarrow 3X$ is possible. However, when considering theories with λ_h , it is essentially impossible to achieve two stage cascades as the dominant decay mode, see figure 3. Furthermore, with λ_h , \tilde{s} is lighter than \tilde{a} due to mixing, making it difficult to arrange a scenario where something of the form $\tilde{h} \rightarrow 2\tilde{s} \rightarrow 4\tilde{a} \rightarrow 4X$ can occur.

However, in the broader context we are considering here, such decays can easily arise, with only moderate spectral tuning. In general, the models rely upon the presence of D-term supersymmetry breaking through the supersoft operator m_D .

There are two forms which we can study, one with CP mixing and one without. With CP mixing, we can consider $\tilde{h} \rightarrow \tilde{a}\tilde{s} \rightarrow 3\tilde{a} \rightarrow 6\tau, 6b$. Such scenarios can easily occur with essentially no tuning, and occur when the decay $\tilde{h} \rightarrow 2\tilde{s}$ is kinematically forbidden. In cases where the six b-quark final state dominates, we also expect a non-trivial fraction of $4b2\tau$ events as well.

Without CP mixing, the cascade generally follows $\tilde{h} \rightarrow 2\tilde{s} \rightarrow 4\tilde{a} \rightarrow 8g, 8\tau, 8b$. Scenarios with so many soft final states have not been studied at LEP, and thus have no constraints beyond the OPAL model independent constraints ($m_h > 82$ GeV). This allows very light stops, and the only tuning required is in achieving the proper spectrum of scalars.

In cases with eight gluon jets, there are typically levels of $6g2\gamma$ which are non-negligible. However, the photons are sufficiently soft that the background is very large, and the jets are very soft, well below the cuts to be applied at the LHC. Likewise, the decays to eight b quarks or eight τ 's seem difficult to study at hadron colliders. These scenarios probably will instead be studied by a future linear collider like the ILC.

Model Building: $\tilde{h} \rightarrow 2\tilde{s} \rightarrow 4\tilde{a} \rightarrow 8g, 8\tau, 8b$ via m_D, A_s, M_Q^{-1}

With the operators m_D, A_s , and M_Q^{-1} , it is possible to find points that have significant

two stage cascades. With the following benchmark point, the Higgs-like mass eigenstate is pushed heavy by mixing with the s , and via dominance of m_D over A_s , decays $\tilde{h} \rightarrow 2\tilde{s}$ are favored over decays into $2\tilde{a}$. The small value of A_s is still sufficient to have \tilde{s} decay into $2\tilde{a}$'s which completes the two stage cascade.

$m_{\tilde{t}}$	δ_s^2	δ_a^2	m_D	A_s	$\sin^2 \theta$	$m_{\tilde{h}}$	$m_{\tilde{s}}$	$m_{\tilde{a}}$	$B(\tilde{h} \rightarrow 2\tilde{a})$	$B(\tilde{h} \rightarrow 2\tilde{s})$	$B(\tilde{s} \rightarrow 2\tilde{a})$	tuning
360	40^2	-52^2	60	8	.06	111	39.3	16.2	.35	.50	.99	4%
		(-54^2)						(7.13)	(0.36)	(0.49)		(2%)

(5.6)

At this point, the constraints that are independent of the \tilde{a} decay products are: 1) decay independent analysis on \tilde{s} which requires $m_{\tilde{s}} \gtrsim 5\text{GeV}$ for the value of $\sin^2 \theta$, 2) SM searches on the \tilde{s} and \tilde{h} which are both satisfied with stronger constraints on \tilde{h} since $\xi_{\text{SM}}^2 = \cos^2 \theta B(\tilde{h} \rightarrow SM) = .14$.

The final state products of \tilde{a} determine what additional constraints apply to this scenario. In the first line of the benchmark, we will consider gluon and b quark final states. With A_h turned off, a nonzero M_Q^{-1} causes it to decay primarily to gluon jets. For this final state, there are no further analyses that constrain this scenario. However, if the \tilde{a} does mix with A^0 by both turning on a small A_h operator and reducing M_Q^{-1} , decays b 's dominate. In this case, the limits apply to the effective ξ_{2b+4b}^2 defined earlier. We find that these are consistent with the new LEP limits, assuming that the analyses are not sensitive to the $8b$ final state (this sensitivity has not yet been determined by any LEP analysis). If \tilde{a} is lighter than 10 GeV, again listed in the second line with new parameters/changes in parentheses, τ 's can dominate, and is consistent with the new LEP analysis of 4τ events from \tilde{s} -strahlung and Higgs-strahlung. Finally, in the case that M_Q^{-1} and mixing decays are comparable, more complicated scenarios are allowed, with many possible final state topologies.

Model Building: $\tilde{h} \rightarrow \tilde{a}\tilde{s} \rightarrow 3\tilde{a} \rightarrow 6b, 6\tau$ via $m_{\text{CP}}^2, m_D, A_s$

In the presence of m_{CP}^2, m_D and A_s , the coefficient of the $\tilde{h}\tilde{s}\tilde{a}$ coupling is

$$\frac{cv}{\sqrt{2}} = \frac{3m_{\text{CP}}^2 v \Delta s_{2\theta}^2 (m_{\tilde{h}}^2 - m_{\tilde{s}}^2)}{8\sqrt{2}m_{\tilde{h}}^2 m_{\tilde{s}}^2} - \frac{m_{\text{CP}}^2 s_{2\theta} g_Y \left((3c_{2\theta} + 1)m_{\tilde{h}}^2 + (1 - 3c_{2\theta})m_{\tilde{s}}^2 \right) m_D}{8m_{\tilde{h}}^2 m_{\tilde{s}}^2} \quad (5.7)$$

$$+ \frac{m_{\text{CP}}^2 A_s s_{2\theta} \left((3c_{2\theta} + 5)m_{\tilde{h}}^2 + (5 - 3c_{2\theta})m_{\tilde{s}}^2 \right)}{2m_{\tilde{h}}^2 m_{\tilde{s}}^2}$$

Where we have taken the decoupling and large $\tan \beta$ limits, as well as the $m_{\tilde{a}} = 0$ limit and at tree-level $\Delta = g_Y^2 + g^2$.

We are most interested in scenarios where $2m_{\tilde{a}} < m_{\tilde{s}} \sim 60\text{GeV}$. Using the perturbative expression for $\sin \theta_{ah}$ (eq. 4.27), we require $\sin^2 \theta_{ah} < 0.015$, which is a typical upper limit for $\tilde{a} \rightarrow 2b$. The constraints on $\tilde{a} \rightarrow 2\tau$ are roughly $\sin^2 \theta_{ah} < .07$ in the kinematically allowed region. Comparing this limit with eq. 4.27, we have (neglecting terms down by $m_{\tilde{s}}^2/m_{\tilde{h}}^2$)

$$m_{\text{CP}}^2 \lesssim .12 m_{\tilde{s}}^2 / (\sin \theta \cos \theta) \quad (5.8)$$

for b quarks, and

$$m_{\text{CP}}^2 \lesssim .26 m_{\tilde{s}}^2 / (\sin \theta \cos \theta) \quad (5.9)$$

for tau decays. Hence, m_{CP}^2 can be $O(m_{\tilde{s}}^2)$, and thus not a small parameter. One can still use the approximate expressions for estimates and intuitive understanding, but for better than $O(1)$ precision, we must calculate exactly.

The presence of A_s is important, both for $\tilde{h} \rightarrow \tilde{a}\tilde{s}$ and $\tilde{s} \rightarrow 2\tilde{a}$ decays. In general, one does not need a large value for A_s , with 4 GeV sufficient in order to secure sufficiently large $\tilde{h} \rightarrow \tilde{a}\tilde{s}$ and $\tilde{s} \rightarrow 2\tilde{a}$ decays.

Let us consider the benchmark point (mass units are in GeV):

$m_{\tilde{t}}$	δ_s^2	δ_a^2	m_D	A_s	m_{CP}^2	$\sin^2 \theta_{sh}$	$\sin^2 \theta_{ah}$	$m_{\tilde{h}}$	$m_{\tilde{s}}$	$m_{\tilde{a}}$	$B_{\tilde{h} \rightarrow \tilde{a}\tilde{s}}$	$B_{\tilde{s} \rightarrow 2\tilde{a}}$	tuning
250	58^2	-10^2	48	6	42^2	0.10	.01	103	67.0	18.4	.70	.91	100%
		(-20^2)							(66.6)	(9.87)	(0.69)	(0.96)	18%

(5.10)

In line one, we consider b quark decays (gluon decays cannot dominate since there is mixing between a and h). This point is consistent with the limits on the effective ξ_{2b+4b}^2 from $4b$ and $2b$ decays. To get to τ decays, one changes the parameters as listed in line two, which satisfies the constraints on 4τ rates. Again, these results assume limited sensitivity at LEP to the 6 final state decays ($6b$ and 6τ).

6. Phenomenology and Benchmark Point Summary

It is remarkable that by simply extending the MSSM to include a singlet superfield, there can be so many modifications to the Higgs phenomenology. Furthermore, many of these modifications do not arise in the NMSSM framework.

To aid in understanding, we have attempted to isolate a few points in parameter space which demonstrate the relevant phenomenology. There are several qualitative issues which are relevant. Let us begin by reviewing our different decay scenarios.

i) $\tilde{h} \rightarrow 2\tilde{a} \rightarrow 4b$: The new LEP limits are noticeably more constraining, and require a certain degree of suppression in the Higgs production and branching ratio of \tilde{a} to b quarks. This can be achieved with $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 300\text{GeV}$ ($O(10 - 15\%)$ tuning). In a crude attempt to combine limits on $2b$ and $4b$ decays, we defined a cumulative ξ_{2b+4b}^2 (in eq. 5.1) and required it to be less than $\sqrt{2}$. This is allowed principally in the “just so” region, where we have essentially saturated the LEP limits, i.e., pushed the parameters so that LEP was nearly sensitive to this scenario. Hence, these regions may be impacted by the finalized LEP limits. Hidden tunings seem to appear when we attempt to use λ_h , the operator used in the NMSSM, to generate the decays because of the large μ term and the subsequent large mixing term with \tilde{s} . Such hidden tunings are smaller in models with the supersoft operator m_D together with A_s .

ii) $\tilde{h} \rightarrow 2\tilde{s}/2\tilde{a} \rightarrow 4\tau$: We have not demonstrated a model which can achieve these decays above the kinematical threshold (i.e., where $\tilde{a} \rightarrow bb$ is not kinematically forbidden). However, we can achieve these decays by tuning the models to the 10 percent level (necessary

to get the \tilde{a} mass into this region). Furthermore, it is troubling that the best available limits in this region (from OPAL [35]) seem to stop at 86 GeV, a theoretical prior due to constraints in two Higgs doublet models which does not apply in cases with singlets. This motivates a reanalysis of the LEP data without the theoretical bias that $m_{\tilde{h}} < 86$ GeV if $m_{\tilde{a}} \lesssim 10$ GeV.

iii) $\tilde{h} \rightarrow 2\tilde{a} \rightarrow 4g$: It is remarkably simple for this decay to dominate under the assumption that A_h is loop suppressed or absent. Such models could be very natural with arbitrarily light stops (subject to direct search limits), and a Higgs with mass $m_{\tilde{h}}$ as light as 82 GeV, the limit from the OPAL model independent analysis [22]. These scenarios can arise easily with either m_D or λ_h , and generally come with associated $\tilde{h} \rightarrow 2\tilde{a} \rightarrow 2g 2\gamma$ decays at the 7×10^{-3} level.

iv) $\tilde{h} \rightarrow \tilde{a}\tilde{s} \rightarrow 3\tilde{a} \rightarrow 6b, 6\tau$: Such a decay can dominate Higgs decays via the inclusion of the m_D operator, incorporating the sensitivity of the Higgs to D-term supersymmetry breaking, but is difficult to engineer with only λ_h (i.e., without D-term breaking). This is an important example of phenomenology which would not occur in the NMSSM, but easily arises within a general operator analysis. Here there is no analysis to exclude this scenario, meaning a Higgs as light as 82 GeV is allowed. In the case that the final state is composed of b-jets, such a scenario can be quite natural. In the case where the final state is composed of τ 's, the tuning is significant, again at the few percent level. Such a scenario would be difficult to detect at the LHC. Finally, although this scenario seems CP-violating, it can be consistent for \tilde{a}, \tilde{s} to be both CP-even and thus there are no additional contributions to CP-violating observables such as edms.

v) $\tilde{h} \rightarrow 2\tilde{s} \rightarrow 4\tilde{a} \rightarrow 8b, 8\tau, 8g$: This scenario, too, can only dominate with D-term breaking, and not within the narrow NMSSM framework. With D-term breaking, it appears to be necessary to have roughly 10% tuning in order to achieve the proper spectrum (with b or g final states - a few percent if the final state is τ), but again a light Higgs (as light as 82 GeV) is allowed (however, in our benchmark, an additional $4b$ rate pushes up the required Higgs mass). Because the final state particles are so soft, it is difficult to envision a scenario in which the LHC could detect this Higgs.

The benchmarks realizing these phenomenologies are summarized in table 2. The benchmark points illustrate the importance of the general operator analysis. Some scenarios are only natural with the presence of D-term SUSY breaking. The gluon final states only occur once we additionally consider the effects of new, heavy fermions. Such effects are typically excluded from NMSSM analyses and yet we see they can generate some of the most interesting phenomenology. Despite this, one should keep in mind that unexplored operators may also generate the same phenomenology, and thus the phenomenology presented should not only be considered in the context of the particular model realizations.

6.1 Possible exclusions in existing data

To the extent that many of these signals have not been explicitly analyzed, one can argue

Case	1	2	3	4	5
$\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}(\text{GeV})$	325	200	175	360	250
$\delta_s^2(\text{GeV})^2$	45 ²	500 ²	80 ²	40 ²	58 ²
$\delta_a^2(\text{GeV})^2$	57 ²	-35 ² (-51 ²)	0 (-27 ²)	-52 ² (-54 ²)	-10 ² (-20 ²)
$m_D(\text{GeV})$	43	0	30	60	48
λ_h	0	0.3	0	0	0
$A_s(\text{GeV})$	-11	0	8	8	6
$A_h(\text{GeV})$	0	0.3	0 (<i>small</i>)	0/ <i>small</i>	0
$m_{\text{CP}}^2(\text{GeV})^2$	0	0	0	0	42 ²
$\sin^2 \theta_{sh}$	0.1	10 ⁻³	.22	0.06	.10
$\sin^2 \theta_{ah}$	0	0	0	0	.01 (.01)
$m_{\tilde{h}}(\text{GeV})$	109	94	94.9	111	103
$m_{\tilde{s}}(\text{GeV})$	73.8	503	76.2	39.3	67.0 (66.6)
$m_{\tilde{a}}(\text{GeV})$	32.6	38.7(11.1)	28.2 (8.37)	16.2 (7.13)	18.4 (9.87)
$B_{\tilde{h} \rightarrow 2\tilde{a}}$	0.86	0.92(0.96)	.92 (.93)	.35 (.36)	.09 (.12)
$B_{\tilde{h} \rightarrow 2\tilde{s}}$	0	0	0	.50 (.49)	[$\tilde{a}\tilde{s}$].70 (.69)
$B_{\tilde{s} \rightarrow 2\tilde{a}}$.99	0	.99	.99 (.99)	.91 (.96)
$a \rightarrow X$	bb	gg	$gg (\tau\tau)$	$gg, bb (\tau\tau)$	$bb (\tau\tau)$
tuning	3%	60%(3%)	100% (10%)	4%(2%)	100% (18%)
pheno	i	iii	ii, iii	v	iv

Table 2: Benchmark Point Summary. Terms in parentheses are variations on the spectrum in order to achieve $\tilde{a} \rightarrow \tau\tau$ decays. When A_h is labeled 0/small, the 0 indicates the presence of nonzero M_Q^{-1} allowing $\tilde{a} \rightarrow gg$ decays, while the “small” is to indicate a small level allowing $\tilde{a} \rightarrow bb$ or $\tilde{a} \rightarrow \tau\tau$ decays, without modifying the spectrum. [$\tilde{a}\tilde{s}$] indicates that the decay is $\tilde{h} \rightarrow \tilde{a}\tilde{s}$ rather than $\tilde{h} \rightarrow 2\tilde{s}$.

that only the model independent bound from OPAL truly limits them. However, once one specifies a given decay mode, it is almost certain that the bounds will improve from the model independent limit.

Going further, extrapolating from existing limits might give us an estimate on the potential exclusions of an actual analysis. For instance, in searches for $h \rightarrow 4b$ in events where the Z decays hadronically, DELPHI and OPAL force the whole event into 4 jets, which is reasonably efficient even though the total process can have more jets in it (up to 6). So since this analysis is akin to the SM 4 jet analysis done for $h \rightarrow 2b, Z \rightarrow$ hadronic, the flavor-independent analyses may be used to estimate the potential limits on $h \rightarrow 4g$. If one assumes that the efficiencies to reconstruct the $4g$ and $4b$ state are the same, and that the background for $4g$ is the same as for $2g$, one can estimate the scale of exclusion that may be possible in existing data. Doing this, it appears that $h \rightarrow 4j$ below 86 GeV has strong exclusion, that interesting constraints could possibly be set for $86 \text{ GeV} < m_{\tilde{h}} < 100 \text{ GeV}$, and for $m_{\tilde{h}} > 100 \text{ GeV}$ it appears unlikely that interesting constraints could be set. All of this is an extremely rough estimate, but suggests additional analyses in the hadronic channels should be done.

Decays of the Higgs to six and eight parton final states are more difficult to estimate, for instance it is not known how sensitive the current $2b$ and $4b$ searches are to actual $6b$ or $8b$ decays. Analyses that attempt to utilize the multi-jet nature of the decays might be useful, but might encounter issues like jet-finding algorithms constructing fake jets in the signal or background (OPAL and DELPHI use the DURHAM algorithm which is known to have such issues). It is also not clear how much b-tagging can help in the cases of multi-b decays.

6.2 Future experiments

It is not clear to what extent these Higgses may evade detection at the LHC. The six and eight parton decays almost certainly will be challenging. Similarly, the decay of the Higgs into four gluon jets is a tremendous challenge at any hadron collider. The decay $h \rightarrow 2a \rightarrow 2g2\gamma$ should be more promising. Especially as loops involving the Higgsinos can amplify this beyond the expected 7×10^{-3} branching ratio, analyses of such scenarios, particularly at the Tevatron, would be highly motivated. The associated $h \rightarrow 4\gamma$ decay has the added signal of seeing both the two a 's and the Higgs that decayed into them. Unfortunately, the expected 10^{-5} branching ratio is probably too small to be seen, so this channel requires either an enhanced rate either through increased branching ratio or Higgs production via colored sparticle decay chains.

7. Summary and Conclusions

One of the strongest predictions of the MSSM is the character of the Higgs boson, both in its mass and its couplings. However, the simplest extension of the MSSM, adding a singlet, can dramatically alter the Higgs boson phenomenology, in particular by introducing a remarkable set of decay scenarios. We have studied, in a relatively general sense, the effects of singlets on the decays of the Higgs boson in supersymmetric theories. There are a number of general points to be gleaned from this analysis.

First of all, the MSSM expectations of the Higgs boson decays can easily be modified. The presence of the singlet opens up a number of cascade decay possibilities, which are much harder to constrain. Secondly, the NMSSM (where the singlet acquires an expectation value to generate the μ -term) is too narrow a framework to realize many of the interesting decay scenarios. A number of them can only dominate the Higgs decay once one includes the possible effects of D-term vevs on the physics of singlets. Some cascades can occur but are remarkably tuned within the NMSSM, but are not tuned in a broader framework. This makes it essential that we continue to include these operators in future analyses.

Finally, while these models admit much lighter Higgses, and hence much lighter stops, than the MSSM, the sensitivity of m_Z is typically inadequate as a measure of tuning. Often the most severe tuning comes from finding an appropriate spectrum such that cascades can occur, while still maintaining a large coupling to the Higgs boson. All analyses of cascade decays must be careful to consider these additional tunings rather than simply focus on the tuning of m_Z . Also, through mixing, heavier Higgses can be allowed for lighter top squarks and hence with less tuning of the m_Z .

Such issues notwithstanding, it is quite straightforward to achieve models which are not tuned, and have a Higgs below the MSSM LEP limit. Cascade decays $\tilde{h} \rightarrow 2\tilde{a} \rightarrow 4b$ allow lighter, more natural Higgses, but at the expense of typically saturating the LEP bounds, a tuning of sorts on its own. Plus, these allowed regions will likely be impacted by the finalized LEP limits. Other cascade decays, in particular $\tilde{h} \rightarrow 2\tilde{a} \rightarrow 4g$ and $\tilde{h} \rightarrow \tilde{a}\tilde{s} \rightarrow 3\tilde{a} \rightarrow 6b$ can occur with very little tuning of the \tilde{a} mass, and light Higgses ($m_{\tilde{h}} > 82$ GeV). Decays with τ final states ($\tilde{h} \rightarrow 2\tilde{a} \rightarrow 4\tau$, $\tilde{h} \rightarrow \tilde{a}\tilde{s} \rightarrow 3\tilde{a} \rightarrow 6\tau$, $\tilde{h} \rightarrow 2\tilde{s} \rightarrow 4\tilde{a} \rightarrow 8\tau$) seem to be quite tuned, since there is no apparent symmetry to explain the very light \tilde{a} (except in case of PQ axion, see footnote 7). On the other hand, two stage cascades $\tilde{h} \rightarrow 2\tilde{s} \rightarrow 4\tilde{a} \rightarrow 8b, 8g$ can occur with moderate tuning. In our realizations, two stage cascades (those with six- and eight parton final states) require D-term breaking while gluon final states arise with new, heavy fields, although there are possibly additional means to produce this phenomenology.

The collider tests of these scenarios are largely unexplored. Firstly, since the model independent decay analysis of OPAL [22] is the dominant constraint for most of our scenarios, it would be extremely useful for a LEP-wide analysis to be carried out. LEP may also have been sensitive to some of the new non-standard decays we have introduced, which motivates further investigation by the LEP Higgs search collaborations on these specific decays. Therefore, we strongly suggest that the following analyses be performed:

- $h \rightarrow$ anything (model independent). Such an exclusion performed by the OPAL analysis is an important consideration for *all* models of non-standard Higgs decays, and thus should be performed by as many LEP collaborations as possible.
- $h \rightarrow$ hadronic (i.e., 4 jet, or multi-parton final states). Given that photonic, leptonic, and two jet decays of the Higgs are already highly constrained, it is worthwhile to close a significant means by which a Higgs might hide, namely in a generic hadronic decay. However, such an analysis may be quite difficult to separate from background, even in events where large amounts of b -flavor is required. The simplest analysis should be an extension of flavor-independent Higgs search to flavor-independent cascades.
- $h \rightarrow 2a \rightarrow 4\tau$ above $m_{\tilde{h}} = 85$ GeV. The theoretical considerations which appear to have truncated the analysis at this mass range do not apply in general models, and likely significant constraints can still be placed above this value.

At the LHC and the Tevatron, the most promising decay channel is the $\tilde{h} \rightarrow 2\tilde{a} \rightarrow 2g 2\gamma$ that is associated with 4 gluon decay. At the Tevatron, such a search may be possible, because the jet threshold is sufficiently low. At the LHC, in this topology, the \tilde{a} photon decay could be seen, although a more careful analysis is needed [36]. Since the two gluon jets would be difficult to measure/choose, this topology does not allow detection of the Higgs. The 4 photon decay is probably at too small of a rate to detect, although this may be increased either by increasing the branching ratio or if Higgses appear in cascade decays of colored superpartners (and thus boosting the overall production rate). Many of the other channels seem even more difficult to search at the LHC, but on a positive note, at the ILC, one should be able to detect the Higgs through the Z recoil method (see for

example [53] and for a linear collider analysis of a SUSY model with nonstandard Higgs decays see [54]).

It should be noted that while many of these scenarios are very difficult in terms of Higgs detection, they are not nightmare scenarios. Quite the contrary, the existence of these decays allows a light Higgs and would naturally be associated with a wealth of light superpartners. This suggests that the proper lesson of LEP is not a need for models with a heavy Higgs, but rather that further thinking about the possibilities of a stealthy Higgs is in order.

Acknowledgments

The authors thank Martin Boonekamp, Radovan Dermisek, Rouven Essig, Ian Hinchliffe, David E. Kaplan, Tom Junk, Amit Lath, Konstantin Matchev, Maxim Perelstein, Andre Sopczak, Matt Strassler, Scott Thomas, Jay Wacker and Scott Willenbrock for useful conversations and correspondence. S.C. would like to thank Christopher Tully for pointing out the presentation at SUSY 05 [18]. The authors would especially like to thank Mark Oreglia for extensive discussions regarding the details of LEP Higgs analyses. P.F. would like to thank the CCPP/NYU for kind hospitality while part of this work was completed. P.F. and N.W. thank Technion where this work was initiated. The authors would also like to thank the Aspen Center for Physics where a portion of this work was completed. The work of S. Chang and N. Weiner was supported by NSF CAREER grant PHY-0449818. P.J. Fox was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the US Department of Energy under contract DE-AC02-05CH11231.

A. Trilinear couplings

m_D — Supersoft Operator

The terms arising from the supersoft operator are

$$\tilde{s}^3 \left[-\frac{c_{2\alpha} s_\theta^2}{4} (g_Y m_D c_\theta) \right] \tag{A.1}$$

$$\tilde{s}^2 \tilde{h} \left[\frac{c_{2\alpha} s_\theta}{4} (g_Y m_D (2c_\theta^2 - s_\theta^2)) \right] \tag{A.2}$$

$$\tilde{s} \tilde{h}^2 \left[-\frac{c_{2\alpha} c_\theta}{4} (g_Y m_D (c_\theta^2 - 2s_\theta^2)) \right] \tag{A.3}$$

$$\tilde{h}^3 \left[-\frac{c_{2\alpha} c_\theta^2}{4} (g_Y m_D s_\theta) \right] \tag{A.4}$$

$$\tilde{h} \tilde{a}^2 \left[-\frac{c_{2\beta} s_\theta}{4} (g_Y m_D s_\phi^2) \right] \tag{A.5}$$

$$\tilde{s} \tilde{a}^2 \left[-\frac{c_{2\beta} c_\theta}{4} (g_Y m_D s_\phi^2) \right] \tag{A.6}$$

while those arising from the D-terms are

D-terms

$$\tilde{s}^3 \left[\frac{c_{2\alpha} s_{\alpha+\beta}}{8} \left(\sqrt{2} \Delta v s_\theta^3 \right) \right] \quad (\text{A.7})$$

$$\tilde{s}^2 \tilde{h} \left[-\frac{c_{2\alpha} s_{\alpha+\beta}}{8} \left(3\sqrt{2} \Delta v s_\theta^2 c_\theta \right) \right] \quad (\text{A.8})$$

$$\tilde{s} \tilde{h}^2 \left[\frac{c_{2\alpha} s_{\alpha+\beta}}{8} \left(3\sqrt{2} \Delta v s_\theta c_\theta^2 \right) \right] \quad (\text{A.9})$$

$$\tilde{h}^3 \left[-\frac{c_{2\alpha} s_{\alpha+\beta}}{8} \left(\sqrt{2} \Delta v c_\theta^3 \right) \right] \quad (\text{A.10})$$

$$\tilde{h} \tilde{a}^2 \left[-\frac{c_{2\beta} s_{\alpha+\beta}}{8} \left(\sqrt{2} \Delta v c_\theta s_\phi^2 \right) \right] \quad (\text{A.11})$$

$$\tilde{s} \tilde{a}^2 \left[\frac{c_{2\beta} s_{\alpha+\beta}}{8} \left(\sqrt{2} \Delta v s_\theta s_\phi^2 \right) \right] \quad (\text{A.12})$$

where at tree-level $\Delta = g_Y^2 + g^2$. Where the notation is as in section 4; θ is the mixing angle between s and h and ϕ is the mixing angle between a and A^0 , from the A_h operator.

Top corrections

The loops from the top induce trilinear terms with an overall coefficient $C = \frac{3y_t^4}{16\pi^2} \times \log(m_{\tilde{t}_1} m_{\tilde{t}_2} / m_t^2)$

$$\tilde{s}^3 \left[\sqrt{2} v s_\theta^3 c_\alpha^3 s_\beta C \right] \quad (\text{A.13})$$

$$\tilde{s}^2 \tilde{h} \left[-3\sqrt{2} v c_\theta s_\theta^2 c_\alpha^3 s_\beta C \right] \quad (\text{A.14})$$

$$\tilde{s} \tilde{h}^2 \left[3\sqrt{2} v c_\theta^2 s_\theta c_\alpha^3 s_\beta C \right] \quad (\text{A.15})$$

$$\tilde{h}^3 \left[-\sqrt{2} v c_\theta^3 c_\alpha^3 s_\beta C \right] \quad (\text{A.16})$$

$$\tilde{h} \tilde{a}^2 \left[-\sqrt{2} v c_\theta c_\alpha s_\beta c_\beta^2 s_\phi^2 C \right] \quad (\text{A.17})$$

$$\tilde{s} \tilde{a}^2 \left[\sqrt{2} v s_\theta c_\alpha s_\beta c_\beta^2 s_\phi^2 C \right] \quad (\text{A.18})$$

$\lambda_h \mathbf{SH}_u \mathbf{H}_d$

The λ_h and μ term give trilinears

$$\tilde{s}^3 \left[-\frac{\lambda_h \tilde{\mu} s_\theta^2 c_\theta}{\sqrt{2}} - \frac{\lambda_h^2 v}{\sqrt{2}} (s_\theta c_\theta^2 s_{\alpha-\beta} + \frac{1}{2} s_\theta^3 s_{2\alpha} c_{\alpha+\beta}) \right] \quad (\text{A.19})$$

$$\tilde{s}^2 \tilde{h} \left[-\frac{\lambda_h \tilde{\mu}}{\sqrt{2}} s_\theta (s_\theta^2 - 2c_\theta^2) - \frac{\lambda_h^2 v}{\sqrt{2}} c_\theta s_{\alpha-\beta} (2s_\theta^2 - c_\theta^2) + \frac{3\lambda_h^2 v}{2\sqrt{2}} s_\theta^2 c_\theta s_{2\alpha} c_{\alpha+\beta} \right] \quad (\text{A.20})$$

$$\tilde{s} \tilde{h}^2 \left[-\frac{\lambda_h \tilde{\mu}}{\sqrt{2}} c_\theta (c_\theta^2 - 2s_\theta^2) - \frac{\lambda_h^2 v}{\sqrt{2}} s_\theta s_{\alpha-\beta} (s_\theta^2 - 2c_\theta^2) - \frac{3\lambda_h^2 v}{2\sqrt{2}} c_\theta^2 s_\theta s_{2\alpha} c_{\alpha+\beta} \right] \quad (\text{A.21})$$

$$\tilde{h}^3 \left[-\frac{\lambda_h \tilde{\mu}}{\sqrt{2}} c_\theta^2 s_\theta + \frac{\lambda_h^2 v}{\sqrt{2}} (s_\theta^2 c_\theta s_{\alpha-\beta} + \frac{1}{2} c_\theta^3 s_{2\alpha} c_{\alpha+\beta}) \right] \quad (\text{A.22})$$

$$\tilde{h} \tilde{a}^2 \left[-\frac{\lambda_h \tilde{\mu}}{\sqrt{2}} s_\theta s_\phi^2 + \frac{\lambda_h^2 v}{\sqrt{2}} (c_\theta c_\phi^2 s_{\alpha-\beta} + c_\theta s_\phi^2 (s_\alpha c_\beta^3 - c_\alpha s_\beta^3)) \right] \quad (\text{A.23})$$

$$\tilde{s} \tilde{a}^2 \left[-\frac{\lambda_h \tilde{\mu}}{\sqrt{2}} c_\theta s_\phi^2 - \frac{\lambda_h^2 v}{\sqrt{2}} (s_\theta c_\phi^2 s_{\alpha-\beta} + s_\theta s_\phi^2 (s_\alpha c_\beta^3 - c_\alpha s_\beta^3)) \right] \quad (\text{A.24})$$

\mathbf{A}_s

In this case the induced trilinears are

$$\tilde{s}^3 \left[-\frac{A_s}{\sqrt{2}} c_\theta^3 \right] \quad (\text{A.25})$$

$$\tilde{h} \tilde{s}^2 \left[-\frac{3}{\sqrt{2}} A_s c_\theta^2 s_\theta \right] \quad (\text{A.26})$$

$$\tilde{h}^2 \tilde{s} \left[-\frac{3}{\sqrt{2}} A_s c_\theta s_\theta^2 \right] \quad (\text{A.27})$$

$$\tilde{h}^3 \left[-\frac{A_s}{\sqrt{2}} s_\theta^3 \right] \quad (\text{A.28})$$

$$\tilde{h} \tilde{a}^2 \left[\frac{3}{\sqrt{2}} A_s s_\theta c_\phi^2 \right] \quad (\text{A.29})$$

$$\tilde{s} \tilde{a}^2 \left[\frac{3}{\sqrt{2}} A_s c_\theta c_\phi^2 \right] \quad (\text{A.30})$$

\mathbf{A}_h

This soft term gives the following trilinears

$$\tilde{s}^3 \left[\frac{A_h}{2\sqrt{2}} s_\theta^2 c_\theta s_{2\alpha} \right] \quad (\text{A.31})$$

$$\tilde{s}^2 \tilde{h} \left[\frac{A_h}{2\sqrt{2}} s_\theta s_{2\alpha} (s_\theta^2 - 2c_\theta^2) \right] \quad (\text{A.32})$$

$$\tilde{h}^2 \tilde{s} \left[-\frac{A_h}{2\sqrt{2}} c_\theta s_{2\alpha} (2s_\theta^2 - c_\theta^2) \right] \quad (\text{A.33})$$

$$\tilde{h}^3 \left[\frac{A_h}{2\sqrt{2}} s_\theta c_\theta^2 s_{2\alpha} \right] \quad (\text{A.34})$$

$$\tilde{h} \tilde{a}^2 \left[\frac{A_h}{\sqrt{2}} s_\phi \left(\frac{1}{2} s_\theta s_\phi s_{2\beta} + c_\theta c_\phi s_{\alpha-\beta} \right) \right] \quad (\text{A.35})$$

$$\tilde{s} \tilde{a}^2 \left[\frac{A_h}{\sqrt{2}} s_\phi \left(\frac{1}{2} c_\theta s_\phi s_{2\beta} - s_\theta c_\phi s_{\alpha-\beta} \right) \right] \quad (\text{A.36})$$

\mathbf{m}_{CP}^2

Since we are ultimately interested in $\tilde{h} \rightarrow \tilde{s} \tilde{a}$ and $\tilde{s} \rightarrow \tilde{a} \tilde{a}$ decays, we find those terms in the Lagrangian. Arising from the usual MSSM D-terms, we have (at leading order in m_{CP}^2),

$$\tilde{a} \tilde{s} \tilde{h} \left[-\frac{3\sqrt{2} m_{\text{CP}}^2 (m_h^2 - m_s^2) v \Delta c_{2\alpha} s_{\alpha+\beta} s_{2\theta}^2}{16(m_s^2 - m_a^2)(m_h^2 - m_a^2)} \right] \quad (\text{A.37})$$

$$\tilde{a}^2 \tilde{s} \left[-\frac{3\sqrt{2} \Delta v m_{\text{CP}}^4 c_{2\alpha} c_\theta^2 s_\theta^3 s_{\alpha+\beta} (m_h^2 - m_s^2)^2}{8(m_s^2 - m_a^2)^2 (m_h^2 - m_a^2)^2} \right] \quad (\text{A.38})$$

While from m_D we have

$$\tilde{a} \tilde{s} \tilde{h} \left[\frac{m_{\text{CP}}^2 g_Y m_D c_{2\alpha} (-2m_a^2 + m_h^2 + m_s^2 + 3(m_h^2 - m_s^2) c_{2\theta}) s_{2\theta}}{8(m_h^2 - m_a^2)(m_s^2 - m_a^2)} \right] \quad (\text{A.39})$$

$$\tilde{a}^2 \tilde{s} \left[-\frac{m_{\text{CP}}^4 g_Y m_D (m_h^2 - m_s^2) c_{2\alpha} s_\theta^2 (-4m_a^2 + 3m_h^2 + m_s^2 + 3(m_h^2 - m_s^2) c_{2\theta}) c_\theta}{8(m_h^2 - m_a^2)^2 (m_s^2 - m_a^2)^2} \right] \quad (\text{A.40})$$

This term is often important in conjunction with A_s . The equivalent trilinears with the combination of A_s and m_{CP}^2 are

$$\tilde{a} \tilde{s} \tilde{h} \left[6A_s m_{\text{CP}}^2 c_\theta s_\theta \left(\frac{c_\theta^2}{m_s^2 - m_a^2} + \frac{s_\theta^2}{m_h^2 - m_a^2} \right) \right] \quad (\text{A.41})$$

$$\tilde{a}^2 \tilde{s} \left[3A_s m_{\text{CP}}^4 c_\theta \left(\frac{c_\theta^2}{m_s^2 - m_a^2} + \frac{s_\theta^2}{m_h^2 - m_a^2} \right)^2 \right] \quad (\text{A.42})$$

References

- [1] Z. Chacko, Y. Nomura and D. Tucker-Smith, *A minimally fine-tuned supersymmetric standard model*, *Nucl. Phys. B* **725** (2005) 207 [[hep-ph/0504095](#)].
- [2] S.P. Martin, *A supersymmetry primer*, [hep-ph/9709356](#).
- [3] P. Fayet, *Supergauge invariant extension of the Higgs mechanism and a model for the electron and its neutrino*, *Nucl. Phys. B* **90** (1975) 104.
- [4] H.P. Nilles, M. Srednicki and D. Wyler, *Weak interaction breakdown induced by supergravity*, *Phys. Lett. B* **120** (1983) 346.
- [5] J.M. Frere, D.R.T. Jones and S. Raby, *Fermion masses and induction of the weak scale by supergravity*, *Nucl. Phys. B* **222** (1983) 11.
- [6] J.P. Derendinger and C.A. Savoy, *Quantum effects and $SU(2) \times U(1)$ breaking in supergravity gauge theories*, *Nucl. Phys. B* **237** (1984) 307.
- [7] L. Durand, J.M. Johnson and J.L. Lopez, *Perturbative unitarity revisited: a new upper bound on the Higgs boson mass*, *Phys. Rev. Lett.* **64** (1990) 1215.
- [8] M. Drees, *Supersymmetric models with extended Higgs sector*, *Int. J. Mod. Phys. A* **4** (1989) 3635.
- [9] J.R. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, *Higgs bosons in a nonminimal supersymmetric model*, *Phys. Rev. D* **39** (1989) 844.
- [10] J.R. Espinosa and M. Quiros, *On Higgs boson masses in nonminimal supersymmetric standard models*, *Phys. Lett. B* **279** (1992) 92.
- [11] P. Batra, A. Delgado, D.E. Kaplan and T.M.P. Tait, *The Higgs mass bound in gauge extensions of the minimal supersymmetric standard model*, *JHEP* **02** (2004) 043 [[hep-ph/0309149](#)].
- [12] P. Batra, A. Delgado, D.E. Kaplan and T.M.P. Tait, *Running into new territory in SUSY parameter space*, *JHEP* **06** (2004) 032 [[hep-ph/0404251](#)].
- [13] A. Maloney, A. Pierce and J.G. Wacker, *D-terms, unification and the Higgs mass*, *JHEP* **06** (2006) 034 [[hep-ph/0409127](#)].
- [14] R. Harnik, G.D. Kribs, D.T. Larson and H. Murayama, *The minimal supersymmetric fat Higgs model*, *Phys. Rev. D* **70** (2004) 015002 [[hep-ph/0311349](#)].

- [15] S. Chang, C. Kilic and R. Mahbubani, *The new fat Higgs: slimmer and more attractive*, *Phys. Rev. D* **71** (2005) 015003 [[hep-ph/0405267](#)].
- [16] A. Delgado and T.M.P. Tait, *A fat Higgs with a fat top*, *JHEP* **07** (2005) 023 [[hep-ph/0504224](#)].
- [17] R. Dermisek and J.F. Gunion, *Escaping the large fine tuning and little hierarchy problems in the next to minimal supersymmetric model and $H \rightarrow aa$ decays*, *Phys. Rev. Lett.* **95** (2005) 041801 [[hep-ph/0502105](#)].
- [18] A. Sopczak, http://susy-2005.dur.ac.uk/PARALLEL/SUSY-HIGGS/MON/sopczak_1ep.pdf parallel Talk at SUSY 05.
- [19] ALEPH, DELPHI, L3 AND OPAL collaborations, and the LEP Working Group for Higgs Boson Searches, *Search for neutral MSSM Higgs bosons at lep. lhwg-note/2005-01*.
- [20] N. Arkani-Hamed, L.J. Hall, H. Murayama, D.R. Smith and N. Weiner, *Small neutrino masses from supersymmetry breaking*, *Phys. Rev. D* **64** (2001) 115011 [[hep-ph/0006312](#)].
- [21] F. Borzumati and Y. Nomura, *Low-scale see-saw mechanisms for light neutrinos*, *Phys. Rev. D* **64** (2001) 053005 [[hep-ph/0007018](#)].
- [22] OPAL collaboration, G. Abbiendi et al., *Decay-mode independent searches for new scalar bosons with the opal detector at LEP*, *Eur. Phys. J. C* **27** (2003) 311 [[hep-ex/0206022](#)].
- [23] LEP WORKING GROUP FOR HIGGS BOSON SEARCHES collaboration, R. Barate et al., *Search for the standard model Higgs boson at LEP*, *Phys. Lett. B* **565** (2003) 61 [[hep-ex/0306033](#)].
- [24] L3 collaboration, P. Achard et al., *Search for an invisibly-decaying Higgs boson at LEP*, *Phys. Lett. B* **609** (2005) 35 [[hep-ex/0501033](#)].
- [25] DELPHI collaboration, J. Abdallah et al., *Searches for invisibly decaying Higgs bosons with the Delphi detector at LEP*, *Eur. Phys. J. C* **32** (2004) 475 [[hep-ex/0401022](#)].
- [26] LEP HIGGS WORKING FOR HIGGS BOSON SEARCHES collaboration, *Searches for invisible Higgs bosons: preliminary combined results using LEP data collected at energies up to 209-GeV*, [hep-ex/0107032](#).
- [27] LEP collaboration, A. Rosca, *Fermiophobic Higgs bosons at LEP*, [hep-ex/0212038](#).
- [28] DELPHI collaboration, J. Abdallah et al., *Flavour independent searches for hadronically decaying neutral Higgs bosons*, *Eur. Phys. J. C* **44** (2005) 147 [[hep-ex/0510022](#)].
- [29] L3 collaboration, P. Achard et al., *Flavour independent search for neutral Higgs bosons at LEP*, *Phys. Lett. B* **583** (2004) 14 [[hep-ex/0402003](#)].
- [30] OPAL collaboration, G. Abbiendi et al., *Flavour independent search for Higgs bosons decaying into hadronic final states in e^+e^- collisions at LEP*, *Phys. Lett. B* **597** (2004) 11 [[hep-ex/0312042](#)].
- [31] ALEPH collaboration, A. Heister et al., *A flavor independent Higgs boson search in e^+e^- collisions at \sqrt{s} up to 209 GeV*, *Phys. Lett. B* **544** (2002) 25 [[hep-ex/0205055](#)].
- [32] LEP HIGGS WORKING GROUP FOR HIGGS BOSON SEARCHES collaboration, *Flavor independent search for hadronically decaying neutral Higgs bosons at LEP*, [hep-ex/0107034](#).
- [33] OPAL collaboration, G. Abbiendi et al., *Search for neutral Higgs boson in CP-conserving and CP-violating MSSM scenarios*, *Eur. Phys. J. C* **37** (2004) 49 [[hep-ex/0406057](#)].

- [34] DELPHI collaboration, J. Abdallah et al., *Searches for neutral Higgs bosons in extended models*, *Eur. Phys. J. C* **38** (2004) 1 [[hep-ex/0410017](#)].
- [35] OPAL collaboration, G. Abbiendi et al., *Search for a low mass CP-odd Higgs boson in e^+e^- collisions with the opal detector at LEP2*, *Eur. Phys. J. C* **27** (2003) 483 [[hep-ex/0209068](#)].
- [36] B.A. Dobrescu, G. Landsberg and K.T. Matchev, *Higgs boson decays to CP-odd scalars at the Tevatron and beyond*, *Phys. Rev. D* **63** (2001) 075003 [[hep-ph/0005308](#)].
- [37] A. Djouadi, J. Kalinowski and M. Spira, *Hdecay: a program for Higgs boson decays in the standard model and its supersymmetric extension*, *Comput. Phys. Commun.* **108** (1998) 56 [[hep-ph/9704448](#)].
- [38] P.J. Fox, A.E. Nelson and N. Weiner, *Dirac gaugino masses and supersoft supersymmetry breaking*, *JHEP* **08** (2002) 035 [[hep-ph/0206096](#)].
- [39] L.M. Carpenter, P.J. Fox and D.E. Kaplan, *The NMSSM, anomaly mediation and a Dirac bino*, [hep-ph/0503093](#).
- [40] M.J. Strassler and K.M. Zurek, *Discovering the Higgs through highly-displaced vertices*, [hep-ph/0605193](#).
- [41] R. Dermisek and J.F. Gunion, *Consistency of lep event excesses with an $H \rightarrow aa$ decay scenario and low-fine-tuning NMSSM models*, *Phys. Rev. D* **73** (2006) 111701 [[hep-ph/0510322](#)].
- [42] V. Ruhlmann-Kleider, *DELPHI 2003-045-CONF-665, DELPHI results on neutral Higgs bosons in MSSM benchmark scenarios*.
- [43] U. Ellwanger, J.F. Gunion and C. Hugonie, *Difficult scenarios for NMSSM Higgs discovery at the LHC*, *JHEP* **07** (2005) 041 [[hep-ph/0503203](#)].
- [44] PARTICLE DATA GROUP collaboration, S. Eidelman et al., *Review of particle physics*, *Phys. Lett. B* **592** (2004) 1.
- [45] A. Pilaftsis and C.E.M. Wagner, *Higgs bosons in the minimal supersymmetric standard model with explicit CP-violation*, *Nucl. Phys. B* **553** (1999) 3 [[hep-ph/9902371](#)].
- [46] M. Carena, J.R. Ellis, A. Pilaftsis and C.E.M. Wagner, *CP-violating MSSM Higgs bosons in the light of LEP2*, *Phys. Lett. B* **495** (2000) 155 [[hep-ph/0009212](#)].
- [47] M. Carena, J.R. Ellis, S. Mrenna, A. Pilaftsis and C.E.M. Wagner, *Collider probes of the MSSM Higgs sector with explicit CP-violation*, *Nucl. Phys. B* **659** (2003) 145 [[hep-ph/0211467](#)].
- [48] B.A. Dobrescu and K.T. Matchev, *Light axion within the next-to-minimal supersymmetric standard model*, *JHEP* **09** (2000) 031 [[hep-ph/0008192](#)].
- [49] L.J. Hall and T. Watari, *Electroweak supersymmetry with an approximate $U(1)(pq)$* , *Phys. Rev. D* **70** (2004) 115001 [[hep-ph/0405109](#)].
- [50] G. Hiller, *b-physics signals of the lightest CP-odd Higgs in the NMSSM at large $\tan\beta$* , *Phys. Rev. D* **70** (2004) 034018 [[hep-ph/0404220](#)].
- [51] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli and L. Randall, *Extranatural inflation*, *Phys. Rev. Lett.* **90** (2003) 221302 [[hep-th/0301218](#)].
- [52] D.E. Kaplan and N.J. Weiner, *Little inflatons and gauge inflation*, *JCAP* **02** (2004) 005 [[hep-ph/0302014](#)].

- [53] P. Janot, *Testing the neutral higgs sector of the mssm with a 300 GeV - 500 GeV e^+e^- collider*, talk given at ICFA Workshop, Saariselka, Finland, Sep 9-14, 1991.
- [54] T. Han, P. Langacker and B. McElrath, *The Higgs sector in a $U(1)'$ extension of the MSSM*, *Phys. Rev. D* **70** (2004) 115006 [[hep-ph/0405244](#)].